Rutgers University – Physics Graduate Qualifying Exam
Thermal Physics – January 16, 2009

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

TP - A

Let the volume V of a substance be a function of P & T : $V = V(P,T)$.

(a) [5 pts] The isothermal compressibility κ_T is defined by: $\kappa_T = -\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T$

The expansion coefficient β is: $\beta = \frac{1}{V}\frac{\partial V}{\partial T}\Big|_P$

Show that $\frac{\partial \beta}{\partial P}\Big|_T = -\left(\frac{\partial \kappa_T}{\partial T}\right)\Big|_P$

(b) [5 pts] The compressibility of copper as a function of temperature at a constant pressure of 1 atm varies linearly from (7×10^{-12} m^2/N) at $T = 0^\circ K$ to (10×10^{-12} m^2/N) at $T = 1000^\circ K$. The expansion coefficient is $\beta = 6\times10^{-5} K^{-1}$ at $T = 800^\circ K$ and at 1atm. Assume $(\partial\beta/\partial P)_T$ is independent of *P*.

Find the amount by which β at $T = 800^\circ K$ changes when the pressure is changed from 1 atm to 1000 atm.

TP - B

Consider a classical ideal gas. Assume *N* molecules in volume *V*. Let *k* be the Boltzmann constant.

(a) [4 pts] Suppose that the constant volume heat capacity per molecule [i.e. the molecular specific heat] is C_V and is known. Derive the expression for the constant pressure heat capacity per molecule [C_P].

(b) [3 pts] For an isothermal process, we know that *PV* = constant, where *P* is the pressure. Derive the expression that replaces it for an adiabatic process.

(c) [3 pts] Suppose that the gas is helium and is contained in a cubic box of side *L*. Suppose the side of the box is halved in an adiabatic process. Assuming that the gas remains governed by the ideal gas laws during this process, calculate the factor by which the pressure increases.

TP - C1

Consider a non-relativistic Fermi gas in two dimensions: ***N*** electrons confined to a square area $A=L^2$.

(a) [4 pts] The density of electronic states in two dimensions per unit area is given by:

$$g^{2D}(\varepsilon) = \frac{m}{\pi\hbar^2}$$

Show how to derive this formula, don't worry about an exact numerical pre-factor, focus on the dependence on energy and mass.

(b) [2 pts] Find the Fermi energy (in terms of ***N*** and A)

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(c) [2 pts] Find the average energy per electron at $T = 0$ (in terms of E_F).

(d) [2 pts] Calculate E_F for the electron density $N/A = 1 \times 10^{16}$ m^{-2} (a typical density of electrons in a field-effect transistor [FET]), assume that the effective mass is $0.2 \times m_e$ (where m_e is the free electron mass). Is the electron gas in an FET degenerate at room temperature?

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TP - C2

Consider a gas where the internal energy U and pressure P are given by:

$$U = \frac{f}{2} N k_B T - b_2 \frac{N^2}{V}$$

$$P = \frac{N k_B T}{V} - b_2 \frac{N^2}{V^2}$$

The N is the number of molecules in a volume V, f the number of degrees of freedom and b_2 is the second virial coefficient (taken here as independent of T).

(a) [4 pts] Find an expression for the entropy as a function of T and V.

(b) [2 pts]Derive an expression for the connection between T and V for an adiabatic process.

(c) [4 pts] Find expressions for C_V, the specific heat per particle at constant volume, and C_p the specific heat per particle at constant pressure.

Helpful hint: $\left.\frac{\partial T}{\partial P}\right|_V \left.\frac{\partial V}{\partial T}\right|_P \left.\frac{\partial P}{\partial V}\right|_T = -1$

TP - D1

The quantum states of a rigid rotator are labeled by the integers l and m where $l \geq 0$ and $|m| \leq l$. In terms of these the energy levels are given by:

$$\varepsilon_{lm} = l(l+1)\tilde{\varepsilon} - m\tilde{\mu}B$$

where $\tilde{\varepsilon}$ is an energy [$\tilde{\varepsilon} = \hbar^2 / 2I$ where I is the moment of inertia], B is the magnetic field, taken to be in the z-direction, while $\tilde{\mu}$ is the z-component of the magnetic moment when $m = 1$. This system is in close thermal contact with a thermal bath at temperature T. Calculate the thermal mean of the magnetic moment μ of the system for

case 1 $\tilde{\mu}B << kT << \tilde{\varepsilon}$ [5 pts]
case 2 $\tilde{\mu}B << \tilde{\varepsilon} << kT$, [5 pts]

where k is the Boltzmann constant. After you obtain your answer for case 2, see if you can obtain a weaker validity condition for the size of B than that given above.

TP - D2

(a) [5 pts] Within the model of isothermal atmosphere, find the ratio of the mass of the atmosphere to the mass of the planet. Assume that the gravitational field is uniform. The acceleration of the free fall, g, and the pressure at the planet's surface, P_o, are known. Calculate this ratio for the Earth. (Hint: $g = GM_{\mathrm{pl}}/R_{\mathrm{pl}}^2$, where M_{pl} and R_{pl} are the mass of the planet and its radius, respectively, the gravitational constant $G = 6.67 \times 10^{-11} \mathrm{Nm^2/kg^2}$).

(b) [3 pts] Find the average potential energy of the molecules in the atmosphere [neglect rotational and vibrational degrees of freedom].

(c) [2 pts] Find the heat capacity per molecule in the atmosphere (don't forget the kinetic energy) [again, neglect rotational and vibrational degrees of freedom].