Rutgers University – Physics Graduate Qualifying Exam
Quantum Mechanics – January 16, 2009

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

QM - A

A particle is described by a wave function $\psi(x)$ which is constant for $|x| < L$ and is zero everywhere else.

a) [2 pts] Use the uncertainty principle to find the most probable range of values of the particle's momentum.
b) [4 pts] Calculate the probability to observe certain value of momentum p_o. Compare with the estimate from (a).
c) [4 pts] What is the wave function of the particle going to be right after the measurement in (b)?

QM - B

A particle of mass m and energy E whose wave function is a one-dimensional plane wave moving in the $+x$ direction is incident on a one-dimensional square well potential given by:

$$V(x) = -|V_o| \text{ for } |x| \le a; \quad \text{and} \quad V(x) = 0 \quad \text{elsewhere}$$

There are certain discreet energies for which the probability density of the transmitted wave (the part of the wave function in the region $x \ge a$) is equal to the probability density of the incident wave. Find an expression for the values of these resonance energies.

[Hint: You can significantly simplify the problem by considering what the amplitude of the reflected wave must be for this case.]

QM - C1

The simplest example of a bound state of two particles is the bound system of an electron and a positron (anti-electron) called positronium.

(a) [3 pts] Using what you know about the hydrogen atom, determine the ground state energy of positronium and the size (average electron-positron separation) of the ground state.

(b) [3 pts] The ground state of positronium decays into a two-photon final state with a lifetime of about 0.1 ns. Find the expression for the polarization part of the coherent two-photon final state. You'll want to use the fact that the ground state of positronium is a state of odd (negative) parity since it is a $1S$ ($L = 0$) state and the intrinsic parity of it's two constituents, the electron and positron, are opposite

(c) [2 pts] Joe and Sarah set up detectors on opposite sides of the positronium to measure the polarization of the emitted photons. If Joe measures the polarization of his photon to be right-handed, what is the probability that Sarah will find her photon to be x-polarized? [Be sure to show your reasoning]

(d) [2 pts] If Joe measures the polarization of his photon to be x-polarized, what is the probability that Sarah will find her photon to also be x-polarized? [Be sure to show your reasoning.]

QM - C2

Consider a particle in the three-dimensional infinite cubical well

$$V(x,y,z) = \begin{cases} 0, & \text{if } 0 < x < a, 0 < y < a, \text{and } 0 < z < a; \\ \infty & \text{otherwise,} \end{cases}$$

(a) [2 pts] Find the stationary state wave function of this particle and its eigen-energies.

(b) [8 pts] Suppose we perturb this potential well by putting a delta function "bump" at the point ($a/4$, $a/2$, $3a/4$):

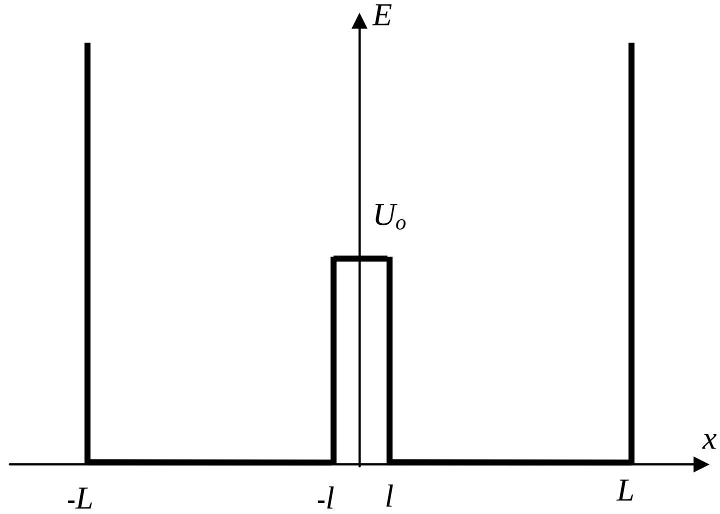
$$V'(x,y,z) = a^3 V_0 \delta\left(x - \frac{a}{4}\right) \delta\left(y - \frac{a}{2}\right) \delta(z - \frac{3a}{4})$$

Find the first-order correction to the energy of the ground state, and the first-order correction to the energy of the triply degenerate first excited states.

QM - D1

A particle moves in the potential shown in the Figure below. Assume that for the low energy levels $E \ll U_o$, but U_o is finite.

a) [3 pts] Sketch the wave function of the first four energy levels.
b) [5 pts] Estimate the energy of the energy levels $E \ll U_0$.
c) [2 pts] Explain the hierarchy of the energy levels and compare it with a particle in a rectangular infinitely deep well.



QM - D2

A hydrogen atom is placed in an electric field $\mathbf{E}(t)$ that is uniform and has the time dependence

$$\mathbf{E}(t) = \begin{cases} 0 & t < 0 \\ \mathbf{E}_0 e^{-\gamma t} & t > 0, \text{ where } \gamma > 0. \end{cases}$$

If the hydrogen atom is initially in the ground state (1*s*), using the first-order time-dependent perturbation theory, find the probability that as t → ∞ the hydrogen atom makes a transition to the triply degenerate 2*p* states.

Here, the relevant radial wave functions ($R_{nl}(r)$) for the hydrogen atom are:

$$R_{10} = 2a^{-3/2}\exp(-r/a) \qquad \text{and;}$$

$$R_{21} = \frac{a^{-3/2}}{\sqrt{24}}\left(\frac{r}{a}\right)\exp(-r/2a)$$

where a represents the Bohr radius, and the energy difference between $2p$ and $1s$ is $\hbar\omega_0$ (>0).