Rutgers University – Physics Graduate Qualifying Exam
## Electricity and Magnetism – January 14, 2009

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

EM - A

Consider a solid spherical shell of inner and outer radii R_1 and R_2, respectively, that has a uniform volume charge density ρ_o.

    (a) [7 pts] Find the electric field as a function of radius, $E(r)$ for all r.
   (b) [3 pts] Plot schematically $E(r)$.
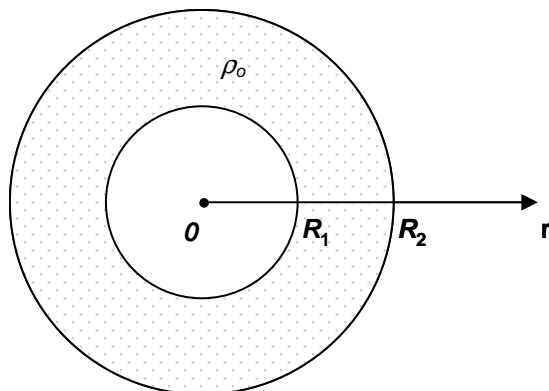
EM - B

A long cylindrical conductor of radius A contains a long cylindrical cavity of radius B $(< A)$ carrying a uniform current density j_o. The axis of the cavity is parallel to the axis of the conductor and the distance between them is $D < A - B$.

Find the magnetic field (magnitude and direction) inside the cavity.

EM - C1

Two semi-infinite grounded conducting plates meet at right angles. In the region between them, there is a point charge q, situated at distances a and b from the planes. Calculate the force on the charge q.
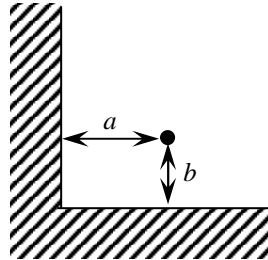

EM - C2

A long, straight, thick wire of radius a has a small gap of width w, such that the ends of the wire form a parallel plate capacitor with circular plates. A current $I(t) = \alpha t$ flows in the wire towards the gap from the left and the same current flows away from the gap towards the right as shown. A rectangular loop of wire of resistance R lies in a plane containing the axis of the thick wire and is located in the gap as shown. Assume all fields are confined to the gap (i.e., ignore fringe fields).

    (a)   [1 pt] What is the surface charge density $+\sigma$ and $-\sigma$, that accumulates on the left and right "plates" of the capacitor as a function of time?

    (b)   [1 pt] What is the electric field in the gap as a function of time (magnitude and direction)?

    (c)   [2 pts] What is the magnetic field in the gap (magnitude and direction)?

    (d)   [3 pts] What is the magnetic flux through the rectangular loop?

    (e)   [2 pts] What is the magnitude of the current induced in the loop?

    (f)   [1 pt]   In what direction does the induced current in the loop flow?
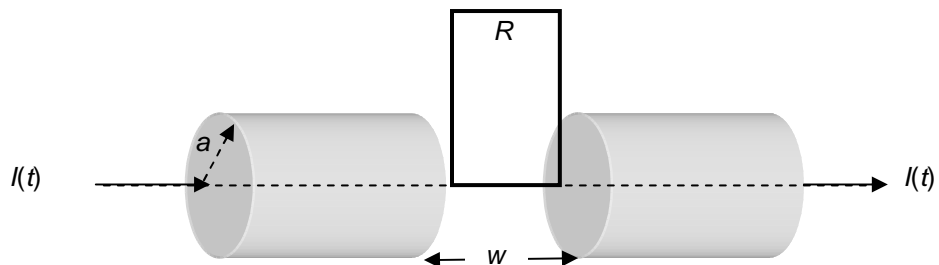
EM - D1

An electrically neutral (uncharged) solid dielectric sphere of radius R with dielectric permittivity ε is placed in a uniform electric field E_0 directed as shown in the Figure (along the y-axis). Find the force acting on a point charge q positioned at a distance $2R$ from the center of the ball. The line connecting the point charge and the center of the ball is perpendicular to the external electric field E_0.
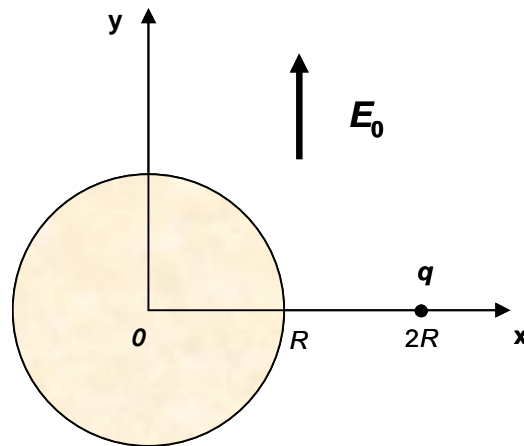
EM - D2

A capacitor is formed by two coaxial cylindrical shells of radii a and b ($a < b$). There is a constant potential difference V between the shells. In the space between them an axially symmetric magnetic field $H(r)$ parallel to the symmetry axis of the cylinders is applied. Here r is the distance to the axis. Electrons are kicked off from the inner shell (that acts as a cathode) with zero initial velocity. At a certain critical value Φ_c of the magnetic field flux through the space between the shells, the magnetic field bends electron trajectories so that they no longer reach the outer shell (the anode). Determine Φ_c. The voltage V is such that the electrons cannot be treated as nonrelativistic particles.