Rutgers University – Physics Graduate Qualifying Exam
Classical Mechanics – January 14, 2009

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

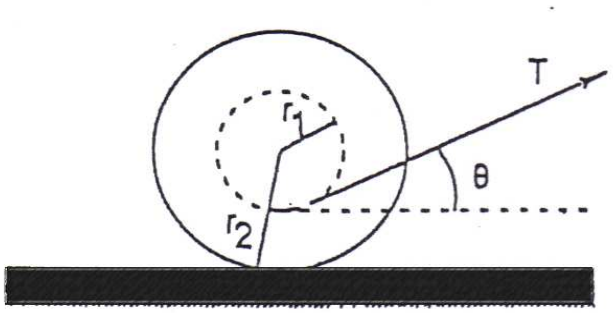
CM - A

A spool rests on a rough horizontal table. A massless thread wound on the spool is pulled with tension T at an angle θ to the horizontal.

(a) [3 pts] If $\theta = 0$ and $r_1 < r_2$, will the spool move to the left or to the right when the thread is slowly pulled? Justify your answer.

(b) [4 pts] Show that there is a critical angle θ_c for which the spool remains at rest. (Assume that T is small enough so that the spool does not slip.)

(c) [3 pts] At this critical angle θ_c, find the maximum tension T_{max} for equilibrium to be maintained (i.e. the spool not to move). Assume a coefficient of static friction μ_s.



CM - B

A rocket with initial mass m_i is initially at rest far from any source of gravitational force. The rocket ejects propellant with a constant velocity u_e with respect to the rocket. Derive the equation for the velocity of the rocket when its final mass is m_f.

CM - C1

A uniform ball of mass *m* rolls without slipping down an incline of mass *M* which itself is sliding down a ramp (see Figure). Assuming the ramp is stationary on a horizontal plane, and neglecting friction between *M* and the ramp, calculate the acceleration of *M*.

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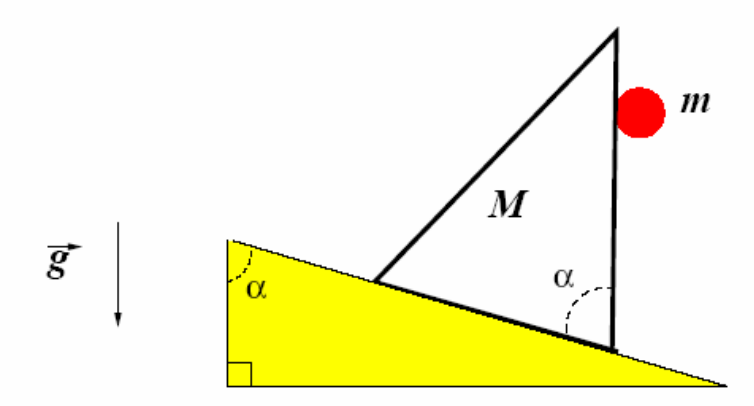
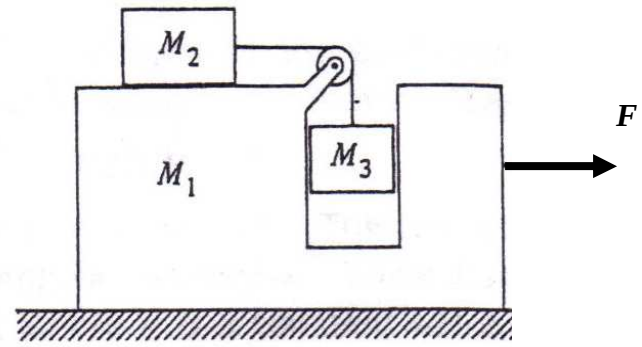


CM - C2

A child's toy is illustrated in the sketch below. All surfaces are frictionless. The object of the toy is for the child to apply a force *F* to M_1 , as shown below, and keep M_3 from rising or falling.

(a) [4 pts] Find the acceleration of M_2.
(b) [6 pts] What is the magnitude of *F*?



CM - D1

A thin, uniform bar of mass M and length $3L/2$ is suspended by a string of length L and negligible mass, as shown below. [Note: The moment of inertia of a thin, uniform bar of length l and mass m about its center of mass, perpendicular to its length is $(1/12)ml^2$.]

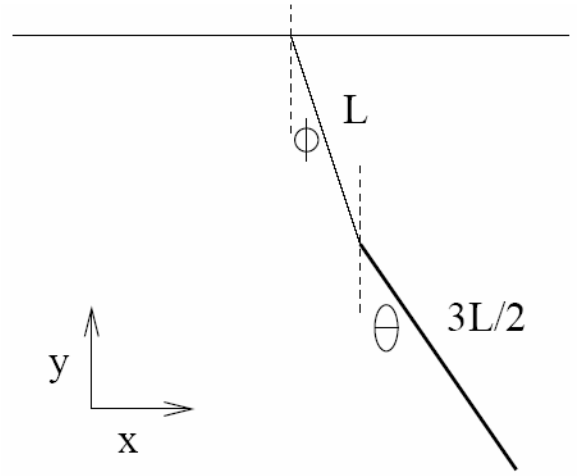
a) [3 pts] In terms of the variables θ and ϕ shown in the figure, what is the position and velocity of the center of mass of the bar in the xy-plane? Write the Lagrangian for arbitrary angles θ and ϕ, and write the Lagrangian appropriate for small oscillations.

b) [2 pts] In the limit of small oscillations, find the Euler-Lagrange equations and show that the equations of motion for the angles θ and ϕ are

$$L\ddot{\theta} + L\ddot{\phi} + g\theta = 0 \quad \text{and} \quad L\ddot{\phi} + \frac{3}{4}L\ddot{\theta} + g\phi = 0$$

c) [2 pts] Again in the limit of small oscillations, write down the form of the normal modes of the system and solve for the frequencies of the normal modes. Describe, both quantitatively and qualitatively, the motion of each normal mode.

d) [3 pts] Now consider the situation where initially the system is at rest with $\theta = \phi = 0$. Starting at time $t = 0$ a constant force of magnitude F is applied horizontally to the bottom of the rod. After a very short time Δt, how are θ and ϕ related?



CM - D2

A system of two pulleys carries a rope as shown in the Figure. The pulley P_{II} is fixed to the ceiling, so that the distance between its center and the ceiling is always h. The pulley P_I is free to move. Each of the pulleys is a solid disk of the radius R and the mass M, the mass being evenly distributed over the disk. The rope has the mass density (the mass per unit length) ρ, and the total length L. At some moment of time the rope is released.

(a) [7 pts] Write down the equation of motion for pulley P_I in terms of its distance from the ceiling.

(b) [3 pts] Find the acceleration of the pulley P_I at the moment of release, if at this moment the center of P_I is at the distance x_o from the ceiling.