Rutgers University – Physics Graduate Qualifying Exam
Thermal Physics – August 31, 2009

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

TP - A

Consider the following process done on an ideal monatomic gas.

1000 joules of heat are added at constant pressure. The pressure is 1 atm.
The system expands in the process and its temperature rises 10 K.

a) [2 pts.] Find the number of moles present.

b) [3 pts.] Find the increase in volume ΔV .

c) [5 pts.] Find the entropy change in terms of the initial temperature T_o.

TP - B

The ground level of the neutral lithium atom is doubly degenerate (that is, $d_0 = 2$). The first
excited level is 6-fold degenerate ($d_1 = 6$) and is at an energy 1.2 eV above the ground level.

(a) [3 pts.] In the outer atmosphere of the Sun, which is at a temperature of about 6000 K, what
fraction of the neutral lithium is in the excited level?   Since all the other levels of Li are at a
much higher energy, it is safe to assume that they are not significantly occupied.

(b) [2 pts.] Find the average energy of Li atom at temperature T
    (again, consider only the ground state and the first excited level).

(c) [5 pts.] Find the contribution of these levels to the specific heat per mole, C_V, and sketch C_V
as a function of T.

TP - C1

The surface of a particular material has a number of equivalent sites at which hydrogen molecules (of mass M) can be adsorbed. Take energy of a hydrogen molecule at one of these sites to be E ($E < 0$). Assume that each of these sites can accept only a single molecule. Assume also that there is no correlation between the absorption probability for a given site on the occupation of any other site. These two facts imply that the occupation number of a site follows a Fermi-Dirac distribution. Take this surface to be in thermal equilibrium with an ideal gas of hydrogen molecules at a common temperature T. Find the occupation probability of a given site as a function pressure P of the gas. Your expression should contain only T, P, E, and fundamental constants. It should be written in a form whose dimensional correctness is self evident. Assume that the hydrogen molecule can be treated as a point mass and that any contribution from any substrate degrees of freedom are negligible.

Useful formulas:

$$\int_0^\infty dx\, x^{2n+1} e^{-x^2} = \frac{n!}{2}$$

$$\int_0^\infty dx\, x^{2n} e^{-x^2} = \frac{1 \cdot 3 \cdot 5 \ldots (2n+1)}{2^{n+1}} \sqrt{\pi}$$

TP - C2

The planet Mercury revolves and rotates at the same rate, so one side of the planet is **always** facing the Sun. Mercury is a distance of 5×10^{10} m from the Sun, and has a radius of 2.44×10^6 m. The radius of the Sun is 7×10^8 m and its total power output is 4×10^{26} W. In this problem treat the planet as if it were a black body.

   a) [1 pt.] What is the energy flux of the Sun's radiation at Mercury's orbit?

   b) [2 pt.] What is the total power absorbed by Mercury?
            [Hint: Consider that it appears as a flat disk to the Sun and it absorbs all of the incident radiation.]

   c)   [5 pts.] If Mercury is in thermodynamic equilibrium, it will emit the same total power as it receives from the Sun. Assuming that the temperature of the "hot" side of Mercury is uniform, find this temperature.

   d) [1 pt.] What is the peak frequency of the radiation absorbed by Mercury?

   e) [1 pt.]   What is the peak frequency of the radiation emitted by Mercury?

TP - D1

A container of volume V is divided by partitions into three compartments containing one kilomole of helium gas, two kilomoles of neon gas, and three kilomoles of argon gas, respectively. Each gas is an ideal gas, with the initial temperature and pressure at $T = 300K$ and $P = 2$ atm. The partitions are removed and the gases diffuse into each other.

a) [2 pts.] What is the final temperature in the system?
What are the final individual pressures of each component and the total pressure?

b) [2 pts.] What is the change in intermal energy?

c) [3 pts.] Calculate the change in Gibbs free energy G in the mixing process.

d) [3 pts.] Calculate the change in entropy of the system in the mixing process.

$R = 8.31$ J/mole-K

TP - D2

Consider a non-relativistic, non-interacting Fermi gas in d dimensions (you may restrict your consideration to $1 \leq d \leq 3$ if you wish). At temperature $T = 0$ the chemical potential of this system is μ_0. Suppose now that T is raised to a small value such that $kT \ll \mu_0$.

How must the chemical potential μ vary so that the density n of the gas does not change?

You should calculate the leading term for the temperature variation of μ at small T. Say qualitatively whether μ increases, remains constant, or decreases with increasing T for various dimensionalities d.

$$\int_0^\infty dx \frac{e^x}{(1+e^x)^2} = \frac{1}{2}$$

$$\int_0^\infty dx \frac{e^x}{(1+e^x)^3} = \frac{1}{6}$$

$$\int_0^\infty dx \frac{e^x}{(1+e^x)^4} = \frac{1}{24}$$

$$\int_0^\infty dx \frac{x^2 e^x}{(1+e^x)^2} = \frac{\pi^2}{6}$$

$$\int_0^\infty dx \frac{x^2 e^x}{(1+e^x)^3} = \frac{\pi^2}{12} - \ln 2$$

$$\int_0^\infty dx \frac{x^2 e^x}{(1+e^x)^4} = \frac{\pi^2}{18} + \frac{1}{6} - \ln 2$$

$$\int_0^\infty dx \frac{x^4 e^x}{(1+e^x)^2} = \frac{7\pi^2}{30}$$