Rutgers University – Physics Graduate Qualifying Exam
Quantum Mechanics – August 31, 2009

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

QM - A

A particle is in a state that is an eigenstate of both total angular momentum and the z-component of angular momentum, $|j,m\rangle$ with $j = 1$ and $m = 1$

a) If the angular momentum of the particle in the z-direction is measured, what are the possible values that can be obtained and what is the probability for each?

b) If the angular momentum of the particle in the *x*-direction is measured, what are the possible value that can be obtained and what is the probability for each?

QM - B

A boy standing on a ladder drops marbles of mass *M* from a height *H*. He tries to hit a point on the ground. Show that even if he is very careful, the marbles are going to miss the point by and average distance Δx which is proportional to $\left(\frac{\hbar}{M}\right)^{1/2}\left(\frac{H}{g}\right)^{1/4}$, where *g* is the gravitational acceleration. How large is the average distance for $M = 1\text{g}$ and $H = 2\text{m}$? Compare with the size of an atom and atomic nucleus.

QM - C1

The angular wave function of a particle is given by $Y(\theta, \phi) = A\sin(\theta)\cos(\phi)$:

(a) [4 points] Find the normalization constant A.

(b) [6 points] Evaluate the expectation values $\langle L_z \rangle$ and $\langle L^2 \rangle$ of this state.

QM - C2

Consider a quantum system that is described by Hamiltonian $H = E_0 \begin{pmatrix} 1 & \lambda \\ \lambda & -1 \end{pmatrix}$. Any wave function $|\psi\rangle$ may be written as a linear combination of the two basis states, $|U\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|D\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

a) Obtain exact energy eigenvalues of this system.

b) Consider the Hamiltonian as $H = H_W + H_V$, where $H_W = E_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $H_V = E_0 \begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix}$, and, assuming that $\lambda << 1$, obtain the energy eigenvalues to second order in perturbation theory. Compare them to exact results and explain the difference.

c) Suppose at $t_0 = 0$ the system is in state $|U\rangle$. What is the probability to find it in state $|D\rangle$ after a time t ?

QM - D1

A particle A with spin 1/2 decays at rest into two particles B and C, where B has spin 1/2 and C has spin 1.

a) What are the possible values of the **orbital** angular momentum of the final state?

b) If the orbital angular momentum of the final state is 0, and if A is polarized with its spin in the $+z$ direction, what is the probability that the z-component of the spin of B will also be in the $+z$ direction? You may find the following Clebsch-Gordon coefficients $\langle j_1, m_1; j_2, m_2 | J, M \rangle$ useful.

QM - D2

An electron is trapped in a one dimensional harmonic potential characterized by the classical angular frequency ω. At time $t = -\infty$ the electron is in the $n = 2$ eigenstate. The system is subjected to a time dependent electric field $E(t)$ that acts from $t = -\infty$ to $t = +\infty$ and is given by

$$E(t) = E_0 e^{-\frac{t^2}{\tau^2}}$$

(a) To first order, what is the probability that the electron will be in the state n' at time $t = -\infty$?
[Note: ignore any interactions associated with the vector potential]

(b) What characteristic time, τ, of the electric field will result in the highest probability that the electron will be found in the state n'?

Possibly useful information:

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{a^2}} e^{ibx}\, dx = a\sqrt{\pi} e^{-\frac{a^2 b^2}{4}}$$