EM – A
Solution:

(a) Apply Gauss' Law to each cavity, with a Gaussian surface just in the conductor, so that the electric field is zero ($\boldsymbol{E}$ = 0). This implies $\oint_i \boldsymbol{E} \cdot d\boldsymbol{a} = 0$ ➔ q_{encl} = O = (pt. charge) + (surface charge). From this:

Surface A: Since pt charge +q inside, charge on surface is –q.
Surface B: Since q = 0 inside, q_{surf} = 0.
Surface C: pt. charge = -q so surface charge is +q.
Surface D: no pt charge so q_{surf} = 0.

(b) Spherical surface with charge –q. From symmetry $\sigma = \frac{-q}{4\pi a^2}$

(c) For surface C we don't have symmetry. We can say that the total charge on this surface is +q but we cannot determine σ without more information about the detailed shape of the surface and the location of the charge within the cavity.

(d) Since surface B contains no charge, and $\boldsymbol{\sigma}_{\text{B}}$ = 0 we have $\boldsymbol{E}$ = 0 at both I and II.

(e) A Gaussian surface just inside the sphere has q_{encl} = +q - q + q = +q . This charge will distribute itself uniformly on the surface regardless of the nature of the cavities in the interior, so $\sigma = \frac{q}{4\pi a R^2}$.

EM – B SOLUTION

The initial (total) charge is $Q - C_1V_o$. After the connection, the potential energy stored in capacitor *1* flows into capacitor 2. Part of the energy dissipates in Joule heating of resistor R.

So, the energy equation is:

$$\frac{d}{dt}\left(\frac{q^2}{2C_1}+\frac{(Q-q)^2}{2C_2}\right)=-I^2R=-\dot{q}^2R$$

Where $Q - q(t)$ is the charge of the second capacitor and $I(t)=\dot{q}$. After differentiating we get the equation:

$$q+\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\cdot q= \frac{Q}{RC_2}$$

In order to make this differential equation easy to integrate, substitute: $z=q-\frac{Q}{C_2}\cdot\frac{C_1C_2}{C_1+C_2}$

Then: $z=\frac{d}{dt}\left(q-\frac{Q}{C_2}\cdot\frac{C_1C_2}{C_1+C_2}\right)+\frac{1}{R}\frac{C_1+C_2}{C_1C_2}\cdot\left(q-\frac{Q}{C_2}\cdot\frac{C_1C_2}{C_1+C_2}\right)=0$ which leads to $\frac{dz}{dt}+\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\cdot z=0$

This is solved by : $z=z_oe^{-\frac{t}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)}$ where $z(t)=q(t)-Q\frac{C_1}{C_1+C_2}$ and $z_o=Q\frac{C_1}{C_1+C_2}$

Therefore,

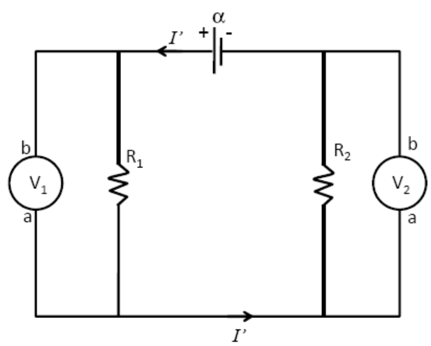
$$I(t)=\dot{z}=\frac{V_o}{R}e^{-\frac{t}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)}$$

EM – C1 SOLUTION

From Faraday's Law:

$$\mathcal{E} = -\frac{d\phi}{dt} = -\alpha$$

The circuit is equivalent to:



The induced current:

$$I' = \frac{\alpha}{R_{tot}} = \frac{\alpha}{R_1 + R_2}$$

We get:

$$V_1 = I'R_1 = \frac{\alpha R_1}{R_1 + R_2}$$

$$V_1 = -I'R_2 = \frac{-\alpha R_2}{R_1 + R_2}$$

EM – C2 SOLUTION

The electric current in the wire is equal to :

$$j_e = en_e(v_e - v_i)$$

Where v_e and v_i are the average velocities of the electrons and ions, respectively. In particular, one sees that there is no current if the velocities are the same.

Let's move to the non inertial coordinate system in which the ions are at rest. Now it is clear that there is a force acting on the electrons in the tangential direction:

$$F_t = m_e \alpha r$$

Where r is the radius of the loop and m_e is the mass of the electron. An effect of such force is equivalent to the tangential electric field:

$$E_t = \frac{F_t}{e} = \frac{m_e \alpha r}{e}$$

As we know, the interaction with the ions does not result in unbounded increase of the electron speed. According to Ohm's law it gives rise to the current:

$$I = \frac{\Delta U}{\Delta R} = \frac{E\Delta l}{\rho \Delta l} = \frac{m_e \alpha r}{e\rho}$$

The magnetic field at the center of the loop is therefore:

$$B = \frac{\mu_o}{4\pi}\frac{2\pi r L}{r^2} = \frac{\mu_o I}{2r} = \frac{\mu_o m_e \alpha}{2e\rho}$$

EM – D1 SOLUTION

(a) The surface charge density is $\sigma = 3\varepsilon_o E_z \cos\theta$. The force acting on a small piece of surface dS is:

$$d\boldsymbol{F} = \frac{\sigma^2}{2\varepsilon_o}\mathbf{n}dS$$

Where $dS = R^2 \sin\theta \, d\theta d\phi$, $\mathbf{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ is the normal vector to the sphere. Hence, the force acting on the upper sphere is:

$$F_{upper} = R^2 \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} d\theta \sin\theta \frac{\sigma^2}{2\varepsilon_o}\mathbf{n}(\phi,\theta) = e_z \pi R^2 (3E_z\varepsilon_o)^2 \int_0^{\frac{\pi}{2}} d\theta \sin\vartheta \cos^3\theta = e_z \frac{9\pi\varepsilon_o}{4} E_z^2 R^2$$

(b) In the case with a total charge Q, the surface charge density is:

$$\sigma = 3\varepsilon_o E_z \cos\theta + \frac{Q}{4\pi R^2}$$

EM – D2 SOLUTIONS

Let $\hat{z} \parallel \boldsymbol{H}$. The equations of motion are:

$$m\frac{du_i}{d\tau} = \frac{e}{c}F_{ik}u_k$$

Where u_k is the particle's velocity, τ is the proper time and F_{ik} is the electromagnetic field tensor.

In the present case, the only nonzero components of F_{ik} are

$$F_{12} = -F_{21} = H$$

Therefore: $\frac{d^2x}{d\tau^2} = \omega_1\frac{dy}{d\tau}$; $\frac{d^2y}{d\tau^2} = -\omega_1\frac{dx}{d\tau}$; $\frac{d^2z}{d\tau^2} = 0$; $\frac{d^2t}{d\tau^2} = 0$

Where $\omega_1 = eH/mc$. Let $w = x + iy$. Then: $\frac{d^2w}{d\tau^2} + i\omega_1\frac{dw}{d\tau} = 0$

From the last equation of (3) we have: $ct = \frac{\mathcal{E}_o}{mc}\tau$ where $\mathcal{E}_o = c\sqrt{p^2 + m^2c^2}$

$$\mathcal{E} = mc^2\frac{dt}{d\tau} = \mathcal{E}_o$$

The energy of the particle, $\mathcal{E}$, is time independent since magnetic forces do no work. Integrating the equations for w and z, separating real and imaginary parts of w, and expressing τ through t, we find:

$$x = R_1\cos(\omega_2 t + \alpha) + \frac{cp_{oy}}{eH} + x_o$$

$$y = -R_1\sin(\omega_2 t + \alpha) - \frac{cp_{oy}}{eH} + y_o$$

$$z = -v_{oz}t$$

We see that the particle moves in a helix about $\boldsymbol{H}$. The radius of the helix is $R = |R_1|$ where:

$$R_1 = \frac{cp_{o\perp}}{eH} \quad \text{and} \quad p_{o\perp} = \sqrt{p_{ox}^2 + p_{oy}^2}$$

The frequency of rotation is: $\omega = |\omega_2|$, where $\omega_2 = eHc/\mathcal{E}$The pitch of the helix is $\frac{2\pi|v_{oz}|}{\omega} = \frac{2\pi\mathcal{E}|v_{oz}|}{|e|Hc}$ where $|v_{oz}| = \frac{p_{oz}c^2}{\mathcal{E}}$.

Finally, the angle a is determined by the following equations: $\sin\alpha = -\frac{p_{ox}}{p_{o\perp}}$, $\cos\alpha = -\frac{p_{oy}}{p_{o\perp}}$