# Rutgers University – Physics Graduate Qualifying Exam
# Classical Mechanics – August 14, 2009

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

## CM - A

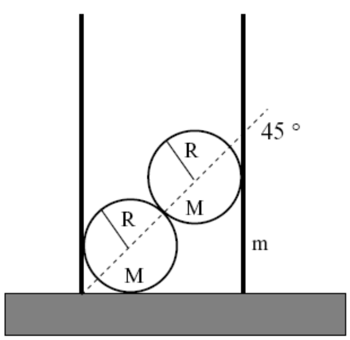
Energetic nuclei |cosmic rays| impact gas atoms at the top of the Earth's atmosphere and create muons with an energy much larger than the muon rest mass. Assume that the muons are all created at the same height and travel straight down towards the Earth's surface

(a) [2points] If the muons are created at analtitude of h=60km, approximately how long does it take them to reach the surface of the Earth? How long is this compared to the laboratory half- life of $t_{1/2} = 1.5 \times 10^{-6}$ s?

(b) [8points] Observations show that 1/8$^{th}$ of the muons reach the ground with out decaying. Derive an expression for the fractional difference of the speed of the muons from the speed of light: 1-b   1 – v/c. You do NOT need to plug numbers into the expression.

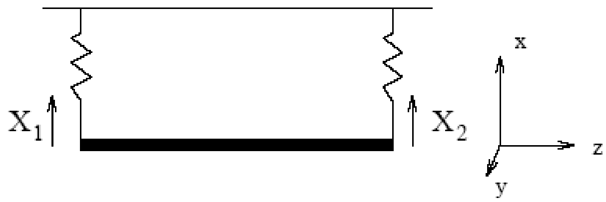
## CM - B

Two steel balls, each of radius *R* and mass *M*, are packed into a bottomless cylindrical thin-walled tube of diameter $L = [2 + \sqrt{2}]$. The tube and balls sit on a table. There is no friction anywhere in the problem. Find the minimum mass of the tube m that will keep the system in equilibrium.



## CM - C1

A rigid uniform bar of mass $M$ and length $L$ is supported in equilibrium in a horizontal position by two massless springs attached at each end.




The identical springs have the force constant $k$. The motion of the center of mass is constrained to move parallel to the vertical $x$-axis. Furthermore, the motion of the bar is constrained to lie in the $xz$-plane. Let $x_1$ and $x_2$ be the departures of the two ends from their equilibrium positions, as shown.

a) [1 point] Show that the moment of inertia for a bar about the $y$-axis through its center of mass is $\frac{1}{12}ML^2$.

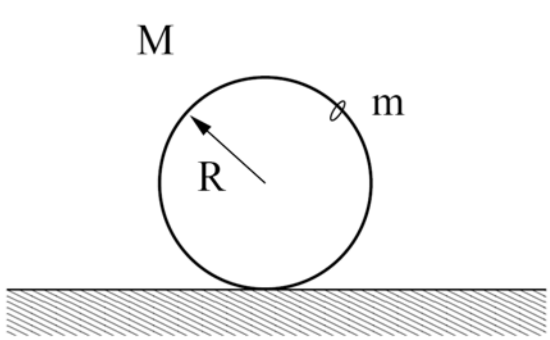
b) [4 points] Construct the Lagrangian for the bar-spring arrangement assuming only small deviations from equilibrium.

c) [4 points] Calculate the vibration frequencies of the normal modes for small-amplitude oscillations.

d) [1 point] Describe the normal modes of oscillation.

## CM - C2

A bead of mass $m$ is constrained to slide along a frictionless circular hoop of radius $R$ and mass $M$. The hoop is free to move along a horizontal frictionless surface. Assuming that the system is initially at rest, find a trajectory of the bead.



## CM - D1

Part I. Consider one-dimensional motion $x(t)$, as described by the Lagrangian:

$$L = \frac{m}{2}\dot{x}^2 - g|x|$$

(a) Write down and integrate the equation of motion.

(b) Find $T(E)$, the period of the motion at given energy $E$.

(c) Find the truncated action per period, $\int_{t_0}^{t_0+T} p\,dx$.

Part II. Same as in Part I, but for the relativistic Lagrangian

$$L = -m\sqrt{1-\dot{x}^2} - g|x|$$

## CM - D2

A pendulum consists of a point mass $m$ suspended by a weightless string wrapped around a fixed cylinder of radius $R$. The length of the string from the mass at rest to the point of tangency is $l$.

(a) Find the Lagrangian of the system.

(b) Using the Lagrangian derive the equation of motion.

(c) Determine the period of oscillation, as a function of energy $E$. Express the result in a form of integral (Don't try to evaluate this integral!).