Rutgers - Physics Graduate Qualifying Exam
Thermal Physics – January 18, 2008

TA

a) [5 pts.] One mole of ideal gas with constant heat capacity C_V is placed inside a cylinder. The cylinder is thermally insulated from the environment; inside the cylinder there is a piston which can move without friction along the vertical axis. Pressure P_1 is applied to the piston; at some point, P_1 is abruptly changed to P_2 (*e.g.* by adding or removing a weight from the piston). As a result, the gas volume changes adiabatically.

Find the temperature T_2 and the volume V_2 after the thermodynamic equilibrium has been reached.

Hint: use the equation of state of ideal gas and the relation between heat capacities C_V and C_P to simplify the formulas

b) [5 pts.] After the thermodynamic equilibrium has been established in part a), the pressure is abruptly reset to its original value P_1. Compute final values of the temperature T_f and the volume V_f after the thermodynamic equilibrium has been reached again. Use the first law of thermodynamics and the adiabatic equation to compute the difference in temperatures (T_f - T_1). Comment on both the sign and the relative magnitude of the temperature difference. What happens in the limit of the vanishingly small change in pressure?

TB

Consider a system of N particles with only 3 possible energy levels separated by ε (let the ground state energy be 0). The system occupies a fixed volume V and is in thermal equilibrium with a reservoir at temperature T. Ignore interactions between particles and assume that Boltzmann statistics applies.

(a) [1 pt.] What is the partition function for a single particle in the system?

(b) [1 pt.] What is the average energy per particle?

(c) [1 pt.] What is probability that the 2ε level is occupied in the high temperature limit, $k_BT >> \varepsilon$? Explain your answer on physical grounds.

(d) [1 pt.] What is the average energy per particle in the high temperature limit, $k_BT >> \varepsilon$?

(e) [1 pt.] At what temperature is the ground state 1.1 times as likely to be occupied as the 2ε level?

(f) [5 pts.] Find the heat capacity of the system, C_V, analyze the low-T ($k_BT << \varepsilon$) and high-T ($k_BT >> \varepsilon$) limits, and sketch C_V as a function of T. Explain your answer on physical grounds.

TC1

Consider a system whose multiplicity (number of accessible states) is described by the equation:

$$\Omega\left(U,V,N\right)=\varphi\left(N\right)V^{N}U^{fN/2}$$

where U is the internal energy, V is the volume, N is the number of particles in the system, N_f is the total number of degrees of freedom, φ(N) is some function of N.

(a) [3 pts.] Find the system's entropy and temperature as functions of U. Are these results in agreement with the equipartition theorem? Does the expression for the entropy makes sense when $T \to 0$?

(b) [2 pts.] Find the heat capacity of the system at fixed volume.

(c) [5 pts.] Assume that the system is divided into two sub-systems, A and B; sub-system A holds energy U_A and volume V_A, while the sub-system B holds $U_B = U\text{-}UA$ and $V_B = V - V_A$. Show that for an equilibrium macropartition, the energy per molecule is the same for both sub-systems.

TC2

The Maxwell-Boltzmann distribution for a collection of monoatomic molecules of mass m (with no internal degrees of freedom) is given by:

$$\Phi\left(x,y,z,p_x,p_y,p_z\right)d\tau=\frac{e^{-\beta E}d\tau}{\int_{-\infty}^{\infty}e^{-\beta E}d\tau}$$

where $d\tau = dxdydxdp_xdp_ydp_z$, $\beta = 1/k_BT$ and the total energy of a molecule is given by:

$$E=(p_x^2+p_x^2+p_x^2)/2m+V\left(x,y,z\right)$$

where p_x; p_y; p_z are the components of the momentum given by mv_x, etc.

a) [3 pts.] Integrate over x, y, and z and show that Eq. (1) reduces to the product of three factors of the form:

$$\Psi\left(v_x\right)dv_x=\left(\frac{\beta m}{2\pi}\right)^{1/2}e^{-\beta mv_x^2/2}dv_x$$

with similar expressions for the velocity components v_y and v_z. Sketch the distribution function $\Psi(v_x)$ as a function of v_x.

b) [2 pts.] Obtain the average kinetic energy associated with each velocity component.

c) [2 pts.] Derive and sketch the velocity distribution $f(v)$ where $v^2=(v_x^2+v_x^2+v_x^2)$. Find the velocity at which $f(v)$ has a maximum.

d) [3 pts.]Apply Eq. (1) to the molecules in an ideal atmosphere to obtain the variation of density with elevation. Assume that Earth is at, the gravitational acceleration is g, and that the temperature is constant.

TD1

A system of N particles of spin S are at temperature T in a volume V. The particles have mass m and are non-relativistic. Neglect statistics-ideal gas limit.

(a) [4 pts.] Use the constraint equation that determines the chemical potential μ, and find μ in terms of $N,V,T\ m$. Use:

$$E = \frac{p^2}{2m} + m$$

(b) [3 pts.] Find an expression for the entropy in terms of $U_k = \frac{3}{2} N k_B T$ and V, N.

(c) [3 pts.] Consider the formation of a deuteron (d) in the early universe via the reaction:

$$p \text{ (proton)} + n \text{ (neutron)} \rightleftarrows d.$$

Write the equilibrium condition for this reaction. Use the equilibrium condition to find the equilibrium ratio $[N_d]\ /\ [N_p]\ [N_n]$ in terms of V, T, and the binding energy of d ($m_{\underline{d}} = m_p + m_n - |BE|$) and other factors.

TD2

When the copper atoms form a crystal lattice with the density of atoms of 8.5 x 10^{28} m^{-3}, each atom donates 1 electron in the conduction band. The density of states for the 3-dimensional Fermi gas is:

$$g^{3D}(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2}$$

where m is the effective mass of the conduction electrons (assumed to be the free electron mass).

(a) [5 pts.] Calculate the *Fermi energy* and the *total energy* of all electrons in the conduction band (per unit volume). Express your answer in eV.

(b) [3 pts.] The electrons participate in the current flow if their energies correspond to the occupancy $f(\varepsilon)$ that is not too close to 1 (no empty states available for the accelerated electrons) and not too small (no electrons to accelerate). At T=300K, calculate the energy interval that is occupied by the electrons that participate in the current flow, assuming that for these electrons the occupancy varies between 0.1 and 0.9.

(c) [2 pts.] Using the assumptions of (b), calculate the ratio N_1/N where N_1 is the number of "current-carrying" electrons, N is the total number of electrons in the conduction band. Assume that within the range where the occupancy varies between 0.1 and 0.9, the occupancy varies linearly with energy (see the Figure), and the density of states is almost energy-independent.