Rutgers - Physics Graduate Qualifying Exam
Electricity and Magnetism – January 16, 2008

EA

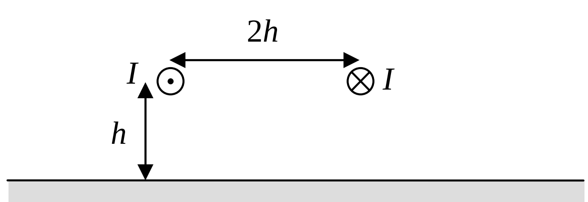
Find the charge density giving rise to the spherically symmetric potential:

$$V(r) = \frac{q}{r} e^{-br}$$

Use Gaussian units. What is the total charge?

EB

Two long, straight wires are parallel to each other and parallel to the surface of a large planar sheet that is a perfect diamagnet (*i.e.*, $\chi_m = -\frac{1}{4\pi}$). The wires are a distance h the surface of the sheet, and $2h$ from each other. Each wire carries a current I, but in opposite directions, as shown in the figure below. Find $\boldsymbol{f}$, the force per unit length on the wire whose current flows into the page.



EC1

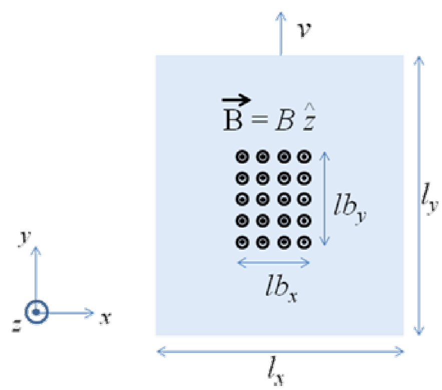
A point charge q is placed a distance $d > R$ from the center of an EQUALLY CHARGED ISOLATED conducting sphere of radius R.

(a) [6 pts] If $d = 2R$, is the point charge attracted or repelled by the sphere?
(b) [4 pts] Repeat part (a) for $d = 1.5\,R$?

Show your work. No points for guessing, but you may find it helpful to consider limiting cases $d >> R$ or $d \sim R$.

EC2

A thin conducting (conductivity g), nonmagnetic sheet of thickness l_z, width l_x and length l_y is moving at speed v in the positive y direction in the presence of a uniform magnetic field of B, which is along positive z direction as shown in the figure. As shown in the figure, the B field is confined to a small rectangular area of $lb_x \times lb_y$ in the middle of the sheet. In this problem we assume $lb_{x,y} << l_{x,y}$.



(a) [3 pts.] If $lb_y << lb_x$, sketch the eddy current and estimate the force and direction exerted onto the moving sheet.

(b) [3 pts.] If the sheet is released at speed v, how long does it take for the speed to drop to half of its initial speed (Take m as the mass of the sheet)?

(c) [3 pts.] Now if $lb_x << lb_y$, [with (lb_x / lb_y) finite] do the same thing as in part (a). (Hint: In this case, the eddy current circuit cannot be well defined, so you should roughly estimate its dimensions based on the sketch of the eddy current. An answer that is within an acceptable numerical factor will receive full credit.)

(d) [1 pts.] You are given a magnet of a rectangular cross-section, $lb_1 \times lb_2$, where $lb_1 >> lb_2$. If you are to design an eddy-current braking system with this magnet, which orientation results in a bigger braking force, that in part 1 or that in part 3? Explain how you come to your conclusion.

ED1

Consider some time dependent charge distribution of finite extent, $\rho(r,t)$, whose time dependent dipole moment is given by $\vec{p}(t) = p_o(t)\,\hat{p}$. In a region of space, the scalar and vector potentials established by this charge distribution are given by:

$$V(\vec{r},t) = \frac{1}{4\pi\varepsilon_o}\left[\frac{Q}{r} + \frac{\hat{r}\cdot\vec{p}(t_r)}{r^2} + \frac{\hat{r}\cdot\dot{\vec{p}}(t_r)}{cr}\right] \quad ; \quad \vec{A}(\vec{r},t) = \frac{\mu_o}{4\pi}\frac{\dot{\vec{p}}(t_r)}{r}$$

where $\dot{\vec{p}}(t)$ is the time derivative of the dipole moment, and $t_r = t - \frac{r}{c}$ is the retarded time.

(a) [5 pts.] Find the electric field $\vec{E}(r,t)$ and magnetic field $\vec{B}(r,t)$ to leading order in powers of $\left(\frac{1}{r}\right)$ in terms of $\ddot{p}_o(t_r)$ [the second time derivative of $p_o(t)$ evaluated at the retarded time, t_r].

(b) [3 pts.] Assume that $\vec{p}(t) = p_o(t)\hat{z}$. Show that $\vec{E}(\vec{r},t) = E(\vec{r},t)\hat{\theta}$ and $\vec{B}(\vec{r},t) = B(\vec{r},t)\hat{\phi}$. Find expressions for $E(\vec{r},t)$ and $B(\vec{r},t)$.

(c) [2 pts.] Find the power radiated to infinity by this time dependent charge distribution.

ED2

A dielectric cylinder of radius a and length L that lies along the z-axis has a polarization given by $\vec{P} = \frac{P_o}{a} s\hat{s}$, in cylindrical coordinates where s is the distance from the z-axis. The cylinder rotates about the z-axis with uniform angular velocity ω. Find an expression for $\vec{B}$ on the z-axis a distance z_o above the top of the cylinder. (some necessary integrals will be given).