Rutgers - Physics Graduate Qualifying Exam
Classical Mechanics – January 16, 2008

MA

Consider a rotating spherical planet of radius R with an equatorial velocity v. The effect of rotation of the planet is to make g at the equator $^2/_3$ of g at the pole. What is the escape velocity for a particle at one of the poles of the planet expressed as a function of v?

MB

Consider a hypothetical planet that is a uniform sphere and has no atmosphere, but otherwise has the same mass and radius R as the earth, and rotates with the same angular velocity ($\omega_e \sim 7 \times 10^{-5}$ rad/sec) from west to east as the earth. During the construction of a tall building located on the equator of this planet, a stone falls directly downward from a crane at a height h = 300 m. Imagine a straight line (a plumb line) from the point of release through the center of the planet.

How far from this line and in which direction with respect to it does the stone strike the surface of the planet?

Answer this question by developing an analytic expression for the horizontal displacement of the impact point from the plumb line which is accurate to linear order in ω_e and assume that $h << R$, and find the order of magnitude of this displacement. Note: g ~ 10 m/s^2.

Hint: This problem is easy, if set up in an inertial (non-rotating) frame of reference.

MC1

A particle of mass m moves under a conservative force with potential energy:

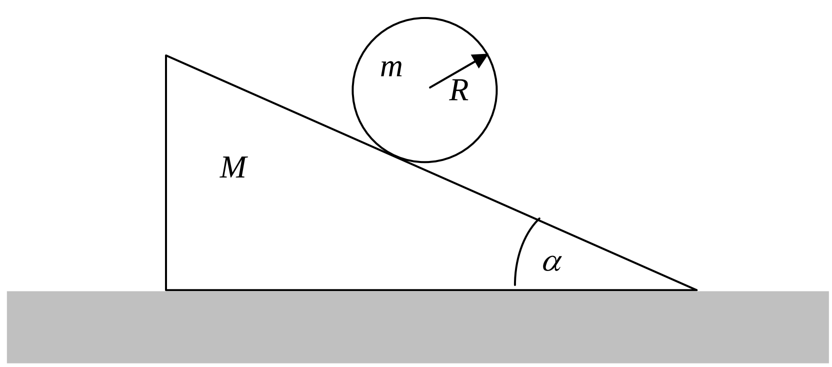
$$V(x) = \frac{cx}{(x^2 + a^2)},$$

where c and a are positive constants.

a) [5 pts.] Find the position of stable equilibrium and the period of small oscillations about it.
b) [5 pts.] If the particle starts from this point with velocity v, find the range of velocity values for which it (1) oscillates, (2) escapes to $-\infty$, and (3) escapes to $+\infty$.

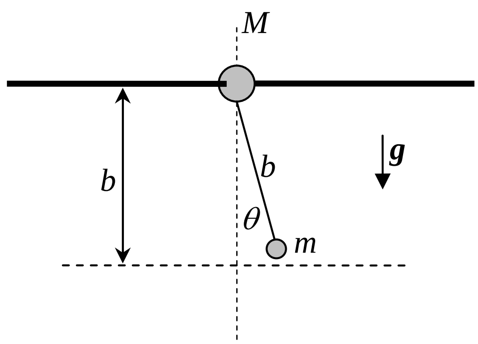
MC2

A uniform ball of mass m and radius R is rolling down without slipping on a wedge of angle α and mass M as shown in the figure. Assuming that the wedge can move without friction on a smooth horizontal surface, calculate its acceleration.



MD1

A bead of mass M slides without friction on a horizontal rod. A second bead of mass m is attached to the first bead with an inextensible massless string of length b, and hangs below under the gravitational force mg downward. We will consider only motions that occur in the vertical plane that intersects the rod. Take a coordinate system whose vertical y-axis intersects the center of the rod, and whose origin is a distance b below the rod, with an x-axis parallel to the rod. Let (x, y) be the coordinates of the mass M, and (x_1, y_1) the coordinates of the mass m. Let θ be the angle of the string with the vertical. See diagram. From the expressions for kinetic energy T and potential energy V, find the normal mode frequencies for small motions away from $x = x_1 = 0$, and quantitatively describe the motions of the two masses corresponding to each normal mode frequency.



MD2

A mechanical model of the ion H_2^+:

Two identical heavy particles (protons) of mass M and the light particle of mass $m << M$ (electron) can move along the straight line only (see Fig.). The "protons" exert constant attractive forces f on the light particle. Furthermore the "electron" is elastically reflected in the collisions with the heavy "protons". Assuming that the size of the ion is a, find a frequency of small oscillations of the heavy particles averaged over the motion of the light one.