

Rutgers University – Physics Graduate Qualifying Exam
Thermal Physics – August 29, 2008

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book, labeled with your code number and the
problem number.
Each problem is worth a total of 10 points.

TP - A

Let H be the enthalpy and let it be dependent on pressure P and temperature T : $H = H(P, T)$.

- a) [4 points] How is H connected to:
- i) the internal energy U and the pressure P and volume V .
 - ii) the Gibbs free energy G , the entropy S , and temperature T .
 - iii) the S , T , the chemical potential μ , and number of particles N

- b) [3 points] What is the relation $\left(\frac{\partial H}{\partial T}\right)_P$ to the heat capacity at constant pressure P .

What is the connection between $\left(\frac{\partial H}{\partial P}\right)_T$ and V , T and the volume expansion coefficient

$$\beta \equiv \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_P. \quad (\text{The number of particles } N \text{ is also held constant.})$$

- c) [3 points] Consider a modified ideal gas equation of state that reads: $P(V - bN) = NkT$

where b is a constant and k is Boltzmann's constant. What is the change in H in an isothermal expansion (at temperature T_1) from volume V_1 to volume V_2 .

TP - B

Consider a system which has energy levels $n\varepsilon$, with $n = 0, 1, 2, 3, \dots$. Six identical spin 1 particles with mass m are placed in these levels so that the total energy is 6ε .

- a) [2 points] List all macrostates or configurations of this system.
- b) [4 points] For each configuration give the counting factor or statistical weight associated with it.
- c) [4 points] Find the mean number of particles in each energy level assuming the system is in equilibrium.

TP - C1

- (a) [5 points] Show by using the density of states that for a photon gas, the pressure is: $P = U/3V$.
- (b) [5 points] Using thermodynamic arguments (First and Second Laws), and the above relationship between pressure and energy density, obtain the dependence of the energy density on the temperature in a photon gas

TP - C2

We want to understand the behavior of non-ideal gases using the canonical ensemble. Let us assume that molecules in the gas interact via inter-molecular interactions $V(r_i - r_j)$, where r_i denotes a 3D radius-vector. There are no intra-molecular interactions. Each molecule has mass m and occupies volume v_c which is inaccessible to all other particles. Approximate the interactions between the molecules by assuming that each particle feels only the average gas density ρ :

$$U_{pot} = \frac{1}{2} \sum_{i \neq j} V(r_i - r_j) \cong Nu_{pot}(v)$$

where $v = V/N = 1/\rho$ (V is the gas volume, and N is the total number of particles). This is known as a molecular field approximation.

Calculate the Helmholtz free energy per particle $f(T, v)$ using a canonical ensemble. Assume that $v_c \ll V$ and $N \gg 1$. Calculate the internal energy and find the equation of state as first derivatives of $f(T, v)$.

Hint: use Stirling's approximation: $\log(N!) \simeq N \log(N) - N + \frac{1}{2} \log(2\pi N)$.

TP - D1

A cubic box (with infinitely hard walls) of volume, $V = L^3$, contains an ideal gas of N identical atoms, each of which has spin S and magnetic moment μ . A magnetic field, \mathbf{B} , is applied to the system.

- (a) [3 points] Compute the partition function for this system.
- (b) [3 points] Compute the internal energy and heat capacity.
- (c) [4 points] What is the magnetization?

TP - D2

A vessel of volume V is filled initially with N non-interacting molecules of type X . U is the total energy of the molecules. The molecules of type X can transform spontaneously into molecules of type Y : $X \Leftrightarrow Y$. This transformation does not require any energy.

- a) [4 points] The total number of microscopic states of this gas, $\Omega(V, N, U)$, is given by:

$$\frac{\Omega(V, N, U)}{\Omega_o(V, N, U)} \equiv \exp\left(\frac{Ns_M^{tot}}{k_B}\right)$$

Find the exact value of s_m^{tot} by summing over all states with different numbers of particles of types X and Y . Show that in the limit of large N

$$\exp\left(\frac{Ns_M^{tot}}{k_B}\right) \approx N \int_0^1 dx \exp\left(\frac{Ns_M(x)}{k_B}\right)$$

where $x = N_X/N$ is the fraction of molecules of type X , Ω_o is the entropy of gas when all molecules are in state X , and $s_M(x)$ is the mixing entropy per particle:

$$s_M(x) = -k_B[x \log x + (1-x) \log(1-x)]$$

- b) [3 points] Find the value of x for which $s_M(x)$ is at its maximum. Use it to find a zero-order approximation to s_m^{tot} . Compare with the exact result from part (a) and explain the differences (if any).
- c) [3 points] Approximate the above integral for $\Omega(V, N, U)$ by the saddle-point method (*i.e.*, compute the Gaussian integral contribution from the second derivative of $s_M(x)$ evaluated at the maximum). Calculate the total mixing entropy s_m^{tot} of the gas under the saddle-point approximation. What is the order of the contributions neglected in part (b)?

Useful formula:
$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} dx = 1$$