Rutgers University – Physics Graduate Qualifying Exam
Quantum Mechanics – August 29, 2008

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book, labeled with your code number and the problem number.
Each problem is worth a total of 10 points.

QM - A

Europium (Eu) (atomic number 63) has a ground state atomic configuration given by:
Eu: (Xe) $4f^7 6s^2$.

(a) [2 points] Explicitly write the full atomic configuration of Eu (eg., $1s^2 2s^2 \ldots 4f^7 6s^2$) where the orbitals are listed in order of their filling.

(b) [2 points] How many electrons can the 5*d* orbital accommodate? Often, the spin-orbit interaction spits the otherwise degenerate 5*d* orbital into non-degenerate levels, indexed according to the quantum number of the total angular momentum, *j*. How many levels does the 5*d* orbital split into, what are the values of *j*, and how many electrons populate each level?

(c) [6 points] Eu in an insulating solid looses three electrons, and becomes an ion. Estimate the magnetic moment associate with the Eu ion. Using the same reasoning, estimate the magnetic moment of a terbium 3+ ion (Tb: (Xe) $4f^9 6s^2$).

QM - B

Consider a particle subject to a constant force $\vec{F}_o$. Show that $\hat{\vec{G}} = \hat{\vec{p}} - \vec{F}_o t$, where $\hat{\vec{p}}$ is the momentum operator and *t* is time, is an operator of a conserved physical quantity, *i.e.*, that the quantum mechanical average is conserved by the evolution. Compare with the corresponding statement in classical mechanics.

QM - C1

A particle of mass m is in the ground state of a one dimensional harmonic oscillator of classical angular frequency ω.

(a) [5 points] Find the normalized eigenfunctions.
(b) [5 points] The angular frequency is suddenly decreased by a factor two. Find the probability that the particle is still in the ground state of the new oscillator.

Useful integral, $\int_{-\infty}^{\infty} e^{-ax^2} dx = \left(\frac{\pi}{a}\right)^{\frac{1}{2}}$

QM - C2

A spin-½ particle with a magnetic moment $\boldsymbol{\mu} = g\, \mu_B \boldsymbol{J}$ ($\boldsymbol{J}$ is the total angular momentum in units of $\hbar$, μ_B is the Bohr magneton) is in a magnetic field $\mathbf{H} = \mathrm{H}\,\hat{z}$. For time $t < 0$ the spin is in the $+z$-direction.

At time $t = 0$, the magnetic field is instantaneously rotated by 90d so that it points in the $+x$-direction.

(a) [4 points] Find the wave function of the particle for all times $t > 0$.

(b) [3 points] Find the expectation value of the J_x, J_y, J_z.

(c) [3 points] If the field is is rotated slowly to the x-direction, taking a total time T, the expectation value of J_x is approximately equal to ½ for all $t > T$. Estimate the shortest time T for which this is a correct description

QM - D1

Determine the eigenstates and eigenvalues of (bosonic) creation and annihilation operators $\hat{a}$ and $\hat{a}^{\dagger}$. Normalize the eignestates where they exist and evaluate the probability distribution of particle (quanta) number *n*, i.e., the probability of finding n particles in the eigenstate. What is the average particle number $\langle n \rangle$?

QM - D2

Consider a particle moving in the attractive spherically symmetric potential:

$$V(r) = -\frac{V_o}{(\frac{r}{a})} e^{-r/a}$$

We wish to explore the properties of the ground state ($l = 0$) energy and wave function that emerge using the variational principle. Assume a ground state wave function of the form:

$$e^{-\beta r/a}$$

Where β is the variational parameter.

(a) [3 points] Find an expression for the ground state energy as a function of the variational parameter β.

(b) [4 points] What is the condition on the relationship between V_o and $\frac{\hbar^2}{2ma^2}$ that must be satisfied to ensure that there is at least one bound solution for $\beta > 0$.

(c) [3 points] Find (but don't bother to solve) the equation for β (> 0) that, if satisfied, would give an upper bound for the ground state energy presuming the condition in part (b) satisfied.