Rutgers - Physics Graduate Qualifying Exam
Mechanics: January 10, 2007

MA

Suppose a block of mass m is projected with initial velocity, v_0, on a smooth horizontal plane, but there is air resistance proportional to v, i.e. $F(v) = -cv$. What is the maximum distance the block will move?

MB

(a) Show that the acceleration of a particle moving in a plane is given by:

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} \quad (1)$$

where r and θ are the usual plane polar coordinates.

(b) A particle moves on a spiral path such that its position is given by:

$$\begin{aligned} r &= bt^2 \\ \theta &= ct \end{aligned}$$

where b and c are constants. Find its velocity and acceleration.

(c) What is unusual about the radial component of the acceleration as t gets large?

MC1

A planet has a mass, M, and radius R. A rocket of mass, m, is launched at an angle, θ, with respect to a radial vector from the center of the planet.

(a) Find the escape velocity for a vertical flight, $\theta = \deg 0$.

(b) Find the circular orbit speed for $r = R$, $\theta = \deg 90$.

(c) Find the maximum height, h (in units of R, i.e. h/R), measured from the center of the planet if the rocket is launched tangentially from the surface with a speed 3/4 the escape speed.

(d) At $r = h$, this rocket is to be put into a circular orbit. How much energy must be added or subtracted to do this?

MC2

Aerodynamic drag forces on an object of mass, m, are given

$$F_D = \frac{1}{2}\rho A C_D v^2, \tag{2}$$

where ρ is the density of air, A is the cross sectional area of the object, C_D is a drag coefficient and v is the speed of the object. Define a constant

$$K = \frac{\rho A C_D}{(2m)} \tag{3}$$

and express your answers in terms of this constant.

(a) A object is constrained to move horizontally subject only to F_D. The object has an initial speed, v_0 and position $x_0 = 0$.

  (i) Find the object's speed at time t.

  (ii) Find the object's position at time t.

(b) The same object is now released vertically from rest in a gravitational field with gravitational acceleration, g. Its initial position is $y_0 = 0$.

  (i) Find the object's speed at time, t, and find its terminal speed.

  (ii) Find the object's position as a function of t.

(c) Check your answers in a-i, a-ii, b-i and b-ii when $C_D = 0$.

Useful formulae:

$$\begin{aligned} \int (\tanh z) dz &= \ln(\cosh z) + c \\ \tanh z &= \{\ln(1+z) - \ln(1-z)\} \end{aligned}$$

MD1

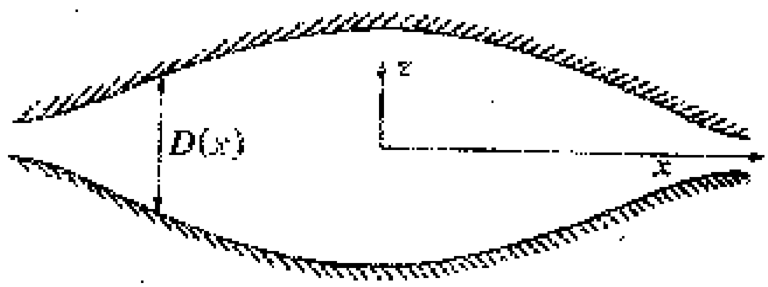
A particle of mass m moves freely between two symmetrical fixed surfaces, as shown below. It bounces off of both surfaces elastically, and the separation betwen the surfaces is given by

$$D(x) = D_0 \left[\frac{\cosh \alpha x}{\cosh 2\alpha x} \right],$$

where D_0 and α are constants.

(a) If $|\dot{z}| >> |\dot{x}|$ for the particle, then, in addition to the total energy, there is another approximate constant of the motion. Write an expression for it.

(b) If the particle starts at the origin in the xz-plane, moving with speed v_0 at a small angle β with respect to the z-axis, find an approximate expression for the value of x_{max}, defined to be the maximum distance the particle will reach in its motion in the x direction, and describe what happens after that.



MD2

Consider a lawn sprayer consisting of a spherical cap subtending an angle of $\alpha_0 = 45^o$ from the vertical, as shown in the picture below. The cap is provided with a large number of spherical holes through which water is ejected with the velocity v_0. The lawn is not uniformly sprayed if these holes are evenly spaced. How must

$$dN/d\alpha,$$

the number of holes per unit angle, be chosen to achieve uniform spraying of a circular area? Assume the radius of the sprinkling cap is very much less than the radius of the area to be sprayed, and the circumference of the cap is at the level of the lawn, as shown.