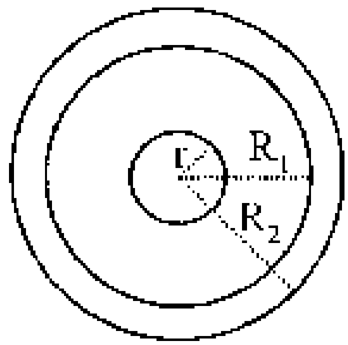
Rutgers - Physics Graduate Qualifying Exam
Electricity and Magnetism: January 10, 2007

EA

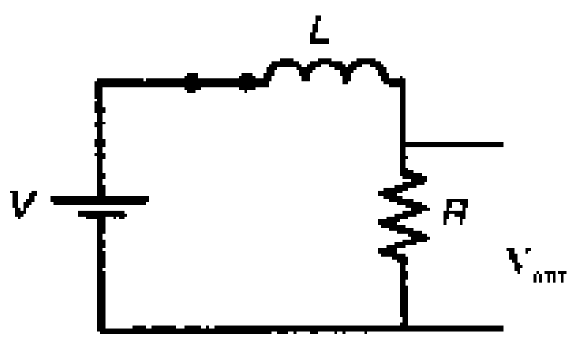
In the following circuit, V has been zero for a while. At the instant when $t = 0$, V suddenly changes to a constant voltage $V_0 \neq 0$. Find out the time dependence of the voltage (V_{out}) across the resistor R. L is an inductor.



EB

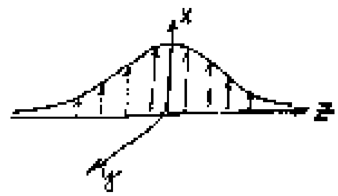
A long, metallic rod with charge Q per unit length and radius r is located on the central axis of a long, metallic cylinder with inner radius R_1 and outer radius R_2. Assume the potential at the central axis of the rod to be zero.

- Write expressions for the electric field as a function of distance from the center of the rod from $r = 0$ to $r = \infty$.
- Write expressions for the electric potential as a function of distance from the center of the rod from $r = 0$ to $r = \infty$.
- Sketch sketch the electric field as a function of r from $r = 0$ to $r = 3R_2$.
- Sketch sketch the electric potential as a function of r from $r = 0$ to $r = 3R_2$.



EC1

A time-dependent, vacuum electromagnetic field in three dimensions, $(\hat{x}, \hat{y}, \hat{z})$ at time, $t = 0$, is shown in the figure.



It has the following form:

$$\begin{aligned}\vec{E}(\vec{r}, t = 0) &= \hat{\imath} E_0 e^{-(z/a)^2} \\ \vec{H}(\vec{r}, t = 0) &= 0\end{aligned}$$

(a) Evaluate and sketch $\frac{\partial \vec{E}(\vec{r},t)}{\partial t}$ at $t = 0$.

(b) Evaluate and sketch $\frac{\partial \vec{H}(\vec{r},t)}{\partial t}$ at $t = 0$.

(c) Evaluate and sketch $\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2}$ at $t = 0$.

(d) What are the values of the fields $\vec{E}(\vec{r}, t)$ and $\vec{H}(\vec{r}, t)$ for a general time, t, satisfying the inequality, $(ct/a) \gg 1$?

(e) Sketch in a single diagram the fields found in (d).

(f) What is the physical significance of the electromagnetic fields appearing in this problem?

EC2

Consider an iron sphere of constant magnetization, M.

(a) Show that $\vec{H}$ can be written, $\vec{H} = -\vec{\bigtriangledown} \phi_m$, where ϕ_m is a scalar function satisfying $\bigtriangledown^2 \phi_m = 0$.

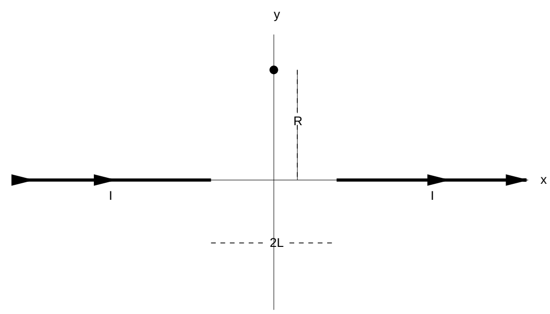
(b) Derive the boundary conditions on ϕ_m.

(c) Find the potential both inside and outside the sphere.

(d) Make three sketches, quantitatively showing $\vec{B}$, $\vec{H}$ and $\vec{M}$.

ED1

A thin wire extends from $x = -\infty$ to $x = -L$ along the x axis, and another from $x = L$ to $x = \infty$, as shown in the figure below. A uniform current of magnitude I flows in the positive x direction in each wire. This means that positive charge is accumulating at a uniform rate at the end ($x = -L$) of the left wire, and negative charge is accumulating at the same rate at the end ($x = L$) of the right wire.



1. Use the circuital version of Maxwell's generalization of Ampere's law to calculate the magnetic field at the point $x = 0$, $y = R$, as designated by the solid dot in the figure.

2. Now use the Biot-Savart law to see what prediction it makes for this field. Is your answer the same as that for the previous part or different? Do you generally expect the Biot-Savart law to give the correct answer for this sort of magnetostatics problem where the displacement current plays a role?

3. Start from the general form of the Biot-Savart law

$$\vec{H}(\vec{r}_1) = \frac{1}{4\pi} \int \vec{j}(\vec{r}_2) \times \frac{\vec{r}_{12}}{|\vec{r}_{12}|^3} \, dV_2, \tag{1}$$

where $\vec{H}$ is the magnetic field, $\vec{j}$ is the current density, and dV_2 is the volume element located at $\vec{r}_2$. Derive the most general form of the Ampere's Law Maxwell equation that follows from it. When is this derived equation valid?

ED2

This problem deals with magnetic braking caused by eddy currents. Consider a *thin* conducting sheet of thickness l_z and conductivity σ lying in the xy plane, that moves in the y direction with a *slow* constant speed v. The sheet lies between the stationary pole pieces of a strong magnet, which produces a magnetic induction B in the $\hat{z}$ direction that is confined to a rectangle between the pole pieces. Said another way, the magnetic field has magnitude B within the rectangle and magnitude zero outside it. Take the rectangle to be centered at the origin, and to have side l_y in the y-direction and side l_x in the x-direction. Assume that the metal sheet has much larger linear dimensions than l_x and l_y. All questions below refer to quantities as measured in the laboratory frame in which the magnet pole pieces are stationary. Assume that the conducting sheet is non-magnetic (relative permeability of unity). Assume $l_z \ll l_x$ and $l_z \ll l_y$.

1. Taking $l_y \sim 1.5 l_x$ make a very rough sketch of the eddy current streamlines that would be expected. Note that outside the rectangle between the pole pieces, these streamlines must coincide with the lines of electric field. Do the eddy current streamlines also coincide with the electric field lines within the above rectangle? Why or why not?
2. Consider the case $l_x \gg l_y$. Calculate the braking force in terms of the given parameters and variables. Show that it is proportional to the area $l_x l_y$ of the pole pieces. Estimate the order of magnitude of this force for a sheet made of a typical conductor like copper or aluminum, when $B = 1$ T, $v = 10$ m/s, $l_x l_y = 1$ cm^2, $l_z = 1$ mm.
3. Consider now the opposite limit of $l_y \gg l_x$. Without trying to obtain this limit analytically, give arguments as to whether the braking force is still proportional to the area of the pole piece footprint, and if not, what the the dependence on this area is.
4. Now consider the case of arbitrary aspect ratio of the magnet footprint. Suppose you had calculated the exact electric field as a function of position. How would you use this information to calculate the current density everywhere and to calculate the the current density everywhere, and to calculate the magnetic braking force? What is called for here are quantitative and unambiguous mathematical expressions, not simply a discussion.
5. Calculate the electric field everywhere.
6. You were asked to answer the previous parts in the limit of a small velocity v. How small is "small"? Describe the essence of the approximation that is appropriate for the small v limit, and estimate the v at which it breaks down.