Rutgers- Physics Graduate Qualifying Exam
Thermodynamics and Statistical Mechanics: August 31, 2007

TA

Black body radiation fills a cavity of volume V. The energy of this radiation is

$$U(T,V) = \frac{4\sigma}{c} V T^4,$$

and the radiation pressure is

$$P = \frac{4\sigma}{3c} T^4.$$

Consider an isentropic (quasi-static and adiabatic) process of cavity expansion. The radiation pressure performs work during the expansion and the temperature of the radiation will drop.

(a) Find how T and V are related for this process.

(b) Assume that the cosmic microwave background (CMB) radiation was decoupled from matter when both were at 3000 K. Currently, the temperature of the CMB radiation is 2.7 K. What was the radius of the universe at the moment of decoupling, compared to now? Consider the process of expansion as isentropic.

TB

Experimental measurements on a mole of gas can be fitted by the equations

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P} + \frac{a}{T^2},$$

$$\left(\frac{\partial V}{\partial P}\right)_T = -T f(P),$$

where V, P, and T are volume, pressure, and temperature, respectively, a is a constant, and $f(P)$ is a function of pressure only. Using these results,

(a) Find $f(P)$.

(b) Find the form of the equation of state of the system.

(c) If the specific heat at constant pressure at low pressures is given by

$$C_P = \frac{5}{2} R,$$

find C_P at arbitrary pressures. You may wish to consider $\left(\frac{\partial C_P}{\partial P}\right)_T$.

TC1 A cylinder is divided into two compartments by a freely sliding piston. Two ideal Fermi gases are placed into the two compartments, numbered 1 and 2. The particles in compartment 1 have spin 1/2, while those in compartment 2 have spin 3/2. They all have the same mass, and move non-relativistically.

(a) Find the equilibrium relative density of the two gases at $T = 0$.

(b) Do the same for the limit $T \to \infty$.

TC2

A rubber band at absolute temperature T is fastened at one end to a peg, and hangs vertically, supporting, at its other end, a large weight, W. To calculate the length of the rubber band, assume, as a simple microscopic model, that it consists of a linked polymer chain of $N >> 1$ oriented segments joined end to end, and that each segment is of length a, and is constrained to be only down, up, or perpendicular to the vertical direction, so that changing a down link to an up one decreases the total length of the chain by an amount $2a$. Neglect the kinetic energies of the segments themselves, and any interactions between the segments other than the fact that they are connected.

(a) Calculate an expression for the mean energy, $< E >$, of the chain in terms of N, a, T, and W.

(b) Use the previous result to calculate the equilibrium length, $< L >$, of the chain. Draw a sketch of $< L >$ as a function of temperature, and discuss why (or why not) this model is reasonable.

TD1

The ground state level of the neutral lithium atom is doubly degenerate. The first excited level is 6-fold degenerate, and is at an energy 1.2 eV above the ground level.

(a) In the outer atmosphere of the sun, which is at a temperature of about 6000 K, what fraction of the neutral lithium is in the first excited level? Since all the other levels of Li are at a much higher energy, it is safe to assume that they are not significantly occupied.

(b) Find the average energy of a Li atom at temperature T (again, consider only the ground state and the first excited level).

(c) Find the contribution of these levels to the specific heat per mole, C_V, and sketch C_V as a function of T. Discuss the curve.

TD2

A satellite, in the form of a cube of edge length L, moves through outer space with a velocity V parallel to one of its edges. The surrounding gas consists of molecules of mass m at temperature T, the number density n of molecules per unit volume being very small, so that the mean free path of the molecules is much larger than L.

(a) Assuming that collisions of the molecules with the satellite are elastic, calculate the mean retarding force exerted on the satellite by the interplanetary gas. You may assume that V is small compared to the mean speed of the gas molecules.

(b) If the mass of the ship is M and it is not subject to any other external forces, after how long a time will of the velocity of the satellite be reduced to half of its original value? Assume $M = 100$ kg, $L = 1$ m, and the satellite is orbiting at a height of 300 km, where the residual gas is mostly O^2 at a density of $n \approx 10^{10}$ cm^{-3}, and $T \approx 10^3$ K.

Note:

$$\int_0^\infty x^n e^{-x}\, dx = n!$$