Rutgers- Physics Graduate Qualifying Exam
Quantum Mechanics: August 31, 2007

QA

Consider the Schroedinger equation for a particle moving in three dimensions in the delta function potential,

$$V(\mathbf{r}) = -a\delta(|\mathbf{r}| - \sigma).$$

(a) Draw a rough qualitative sketch of the lowest energy bound state wave function in this potential.

(b) Find the minimum value of a that would result in a bound state.

QB

Consider an electron in the ground state of a tritium atom (H^3). The tritium now suddenly β-decays to a singly ionized helium-3 (He^3). Assume that both nuclei have infinite mass and that there is no interaction between the β-decay electron and the rest of the system. What is the probability that this new atom will be found in its ground state? Note that the ground state wave function of a one electron atom with nuclear charge Ze is proportional to

$$\psi \propto e^{-Zr/a_0},$$

where a_0 is the Bohr radius, and

$$\int_0^\infty x^n e^{-x}\, dx = n!$$

QC1

(a) Consider the Hamiltonian

$$H_a = \frac{p^2}{2M} + \frac{1}{2}M\omega^2 x^2 + qEx,$$

which characterizes the motion of a charged particle in a harmonic oscillator potential *and* a uniform electric field. Find the ground state eigenvalue and the corresponding wave function.

(b) Consider the Hamiltonian

$$H_b = \frac{p_1^2}{2M} + \frac{p_2^2}{2M} + \frac{1}{2}M\omega^2 x_1^2 + \frac{1}{2}M\omega^2 x_2^2 + \lambda M(x_1 - x_2)^2,$$

which characterizes two coupled harmonic oscillators. Find the ground state eigenvalue and the corresponding wave function. [Hint: Perform simple transformations to bring these Hamiltonians into a familiar form].

QC2

A particle moving in one dimension is confined to an infinite square well, of width L. At $t = 0$, its wave function is piecewise linear and symmetric about $L/2$ with a maximum height of A, as shown in the figure.

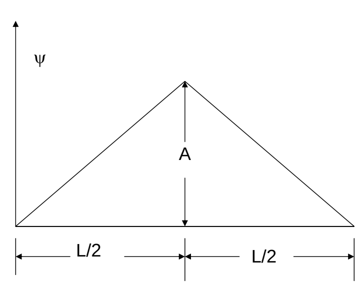
(a) Find A.

(b) Write down the normalized eigenfunctions in this potential.

(c) Expand $\psi(x, 0)$ in terms of the eigenfunctions of part (b).

(d) Write an expression for $\psi(x, t)$. How much time, T, must elapse in order for $\psi(x, T) = \psi(x, 0)$?

(e) Write down an expression of $< E >$, the expectation value of the particle's energy in this state.



QD1

Consider a composite system made up of two spin 1/2 particles. For $t < 0$, the Hamiltonian does not depend on time and can be taken to be zero. For $t > 0$, the Hamiltonian is given by

$$H = \left(\frac{4\Delta}{\hbar^2}\right) \mathbf{S_1} \cdot \mathbf{S_2}$$

where Δ is a constant. Suppose the system is in the state $|+,- > \equiv |\alpha_1, \beta_2 >$ for $t \leq 0$. Find, as a function of time, the probability for being in each of the states $|\alpha_1, \alpha_2 >, |\alpha_1, \beta_2 >, |\beta_1, \alpha_2 >$, and $|\beta_1, \beta_2 >$:

(a) by solving the problem exactly, using $|\Psi(t) >= U(t, t_0)|\Psi(t_0) >$.(b) by solving the problem assuming the validity of first-order time dependent perturbation theory with H as a perturbation which is switched on at $t = 0$.

(c) Under what condition does the perturbation calculation disagree with the exact solution, and why?

QD2

A spinless particle is bound to a fixed center by a potential $V(\mathbf{r})$ which is so asymmetrical that no energy level is degenerate. Denote its energy eigenstates by $|n >$. Assume that the total Hamiltonian is time reversal invariant.

(a) What is the effect of time reversal on $|n >$? In other words, if Θ is the time reversal operator, what is $\Theta|n >$.

(b) What is the effect of time reversal on the spatial part of the wave function corresponding to $|n >$.

(c) Use the preceding results to argue that in any of the energy eigenstates, $< \mathbf{L} >= 0$, where $\mathbf{L}$ is the angular momentum operator. Note that $< L^2 >$ is not necessarily zero for these eigenstates.