Rutgers- Physics Graduate Qualifying Exam
Mechanics: August 29, 2007

MA

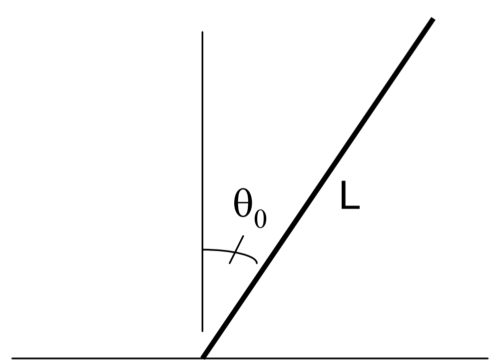
An air molecule is roughly spherical, with a radius R_{air} of about 2×10^{-10} meters. The number of such molecules per unit volume is about $n = 3 \times 10^{25}/\text{m}^3$.

(a) On the average, how far does an air molecule travel between collisions with other such molecules?

(b) How does this compare with the average separation between molecules?

MB

A thin uniform rod of mass m and length L, with its bottom end resting on a frictionless table, is released from rest at angle θ_0 relative to the vertical, as shown in the diagram. Find the force exerted by the table upon the stick at an infinitesimally small time after its release.



MC1

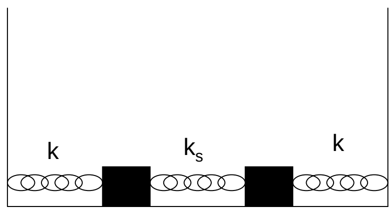
Two masses lie on top of a frictionless plane. Each has mass m, and they are separately connected to parallel walls by springs with spring constant k. In addition, they are connected to each other by a third middle spring with spring constant k_s, as shown in the diagram.

(a) Write equations of motion for each mass.

(b) Find the normal mode frequencies for this system.

(c) Discuss and sketch the behavior of the normal modes.

(c) The left most mass is displaced from its equilibrium position at time $t = 0$ by an amount x_1, while the right hand mass is at its equilibrium position ($x_2 = 0$) at that instant. Find the positions of the two masses at subsequent times.



MC2

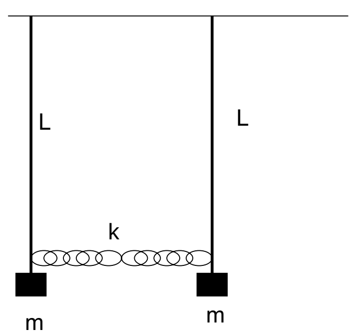
Two pendulums of length L have masses m attached at their ends. They are also connected to each other by a spring with spring constant k, as shown in the figure. The equilibrium positions of the springs are vertical. Assume small displacements.

(a) When each pendulum is displaced in angle find the torque on each. Write equations of motion for each pendulum.

(b) Find the normal mode frequencies of the system.

(c) Discuss and sketch the behavior of the normal modes.

(d) At $t = 0$, one of the pendulums is displaced by an angle θ_1, while the other remains at its vertical position. Find the subsequent angular position of each as a function of time.



MD1

Suppose a spacecraft of mass m_0 and cross-sectional area A is coasting with speed v_0 in the positive x-direction when it encounters a stationary dust cloud of density ρ, at $x = 0$. Assume that the dust sticks to its surface and that A is constant over time.

(a) Find an expression for the spacecraft's velocity, $v(x)$, as a function of the distance it has penetrated into the cloud.

(b) Use the result of part(a) to find the spacecraft's position, $x(t)$, as a function of time. Does this agree with the expected result in the limit $\rho \to 0$?

MD2

If the solar system were immersed in a uniformly dense spherical cloud of weakly-interacting massive particles (WIMPS), then objects in the solar system would experience gravitational forces from both the sun and the cloud of WIMPS such that

$$F_r = -\frac{k}{r^2} - br.$$

Assume that the extra force due to the WIMPS is very small (i.e., $b << k/r^3$).

(a) Find the frequency of radial oscillations for a nearly circular orbit and the rate of precession of the perihelion of this orbit.

(b) Describe the shapes of the orbits when r is large enough so that

$$F_r \approx -br.$$