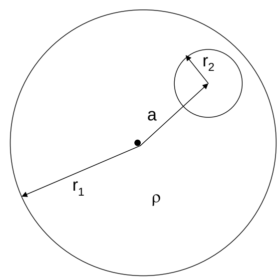
Rutgers- Physics Graduate Qualifying Exam
Electricity and Magnetism: August 29, 2007

EA

A sphere of radius r_1 has has a uniform charge density ρ inside, except for a small spherical hollow region of radius r_2 located a distance a from the center, as shown in the figure.

(a) Find the electric field, $\mathbf{E}$ (both magnitude and direction) at the center of the hollow sphere.

(b) Find the electrostatic potential, V at the same point.

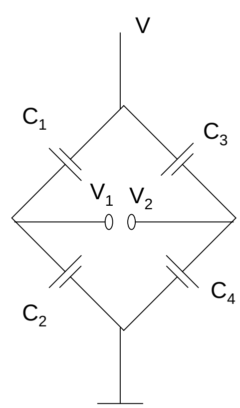


EB

Consider the bridge circuit with capacitors as shown in the figure.

(a) Express the quantities V_1 and V_2 in terms of C_1, C_2, C_3, and C_4.

(b) Find the null condition, i.e., the condition for $V_1 = V_2$.



EC1 Consider a system in which n free electrons, each with charge $-e$, and mass m, are initially distributed uniformly throughout a spherical volume of radius R. Also present in the *same* volume is an equal *uniform* density of ions, each which charge $+e$. Assume that the ions are massive enough that they can be considered to remain permanently at rest; i.e., they are a *rigid* charge distribution.

(a) Find the electric field, $\mathbf{E}$, immediately after all of the free electons are given inifinitesimal **radial** outward displacements proportional to the initial distances of the electrons from the center of the sphere.

(b) Make appropriate approximations to show that, if the electrons are simulataneously released, they will all oscillate radially with the same frequency.

(c) Find the value of that frequency.

EC2

(a) Define the magnetic dipole moment of an arbitrary current distribution given by current density, $\mathbf{j}$.

(b) What is the magnetic dipole moment of a ring of radius a carrying total current, i?

(c) Suppose you have a particle with charge q and mass m, moving with angular momentum $\mathbf{L}$ in a circular orbit, with speed v. Compute the magnetic dipole moment of this system.

(d) Find the precession frequency of the orbit in a magnetic field, $\mathbf{B}$. Express your answer in terms of q, m, and B.

(e) Discuss what is meant by diamagnetism. Give an example of a diamagnetic substance.

(f) Discuss what is meant by paramagnetism. Give an example of a paramagnetic substance.

ED1

At time $t = 0$, a non relativistic charged particle moves with velocity v perpendicular to a magnetic field, $\mathbf{B}$.

(a) Write an expression for the time scale of this motion due to radiative damping.

(b) Find an approximate expression for the particle's trajectory as a function of time.

(c) Assume that this particle is moving through a gas of atoms with density n, and scattering cross section, σ. What is the upper limit on the density, n_{max}, which would be allowed in order for the motion described in (a) and (b) to be observed?

(d) Estimate the order of magnitude of n_{max} for an electron moving with velocity 0.1 c in the earth's atmosphere. (You may assume that σ is the geometrical cross section).

Hint: You will need the formula for the energy radiated by an accelerated charged particle. If you have forgotten this formula, you may approximate it by dimensional analysis. The radiated power is a function of the charge, e, the speed of light, c, and the magnitude of the acceleration, a.

ED2

Find the force between two permanent magnets that have the shape of long cylinders (with length $L >> R$) with magnetization M parallel to the axes of the cylinders, which are positioned coaxially one at the top of the other at a distance $d << R$, as shown in the figure. Assume that ferromagnetic material has the permeability of the vacuum.