Qualifying Examination
Classical Mechanics
January 12, 2006

Problem MA

A block of mass M_1 rests on a block of mass M_2 which lies on a frictionless table, see Fig. MA1. The coefficient of friction between the blocks is μ. What is the maximum horizontal force which can be applied to the blocks for them to accelerate without slipping on one another if the force is applied to (a) block 1, and (b) block 2?

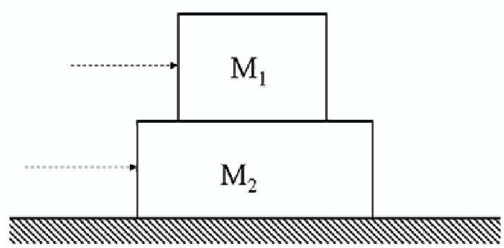

Figure 1.

Problem MB

A commonly used potential energy function to describe the interaction between two atoms is the Lennard-Jones 6-12 potential, $U=\varepsilon[(r_o/r)^{12}-2(r_o/r)^6]$.

(a) Show that the radius at the potential minimum is r_o, and that the depth of the potential well is ε.

(b) Find the frequency of small oscillations about equilibrium for two identical atoms of mass m bound to each other by the Lennard-Jones interaction.

Problem MC1.

When the flattening of the Earth at the poles is taken into account, it is found that the gravitational potential energy of a mass m a distance r from the center of the Earth is approximately $U(r) = -GM_e m r^{-1}[1-0.00054(R_e/r)^2(3\cos^2\theta-1)]$, where θ is measured from the pole (see Fig. MC2). R_e and M_e are the Earth's radius and mass, respectively. Show that there is a small tangential gravitational force on m except above the poles or the equator and find the magnitude of this force as function of r and θ. Find the numerical ratio of this force to the "round-Earth force" $GM_e m/r^2$ for $\theta=45^{\circ}$ and $r=R_e$.

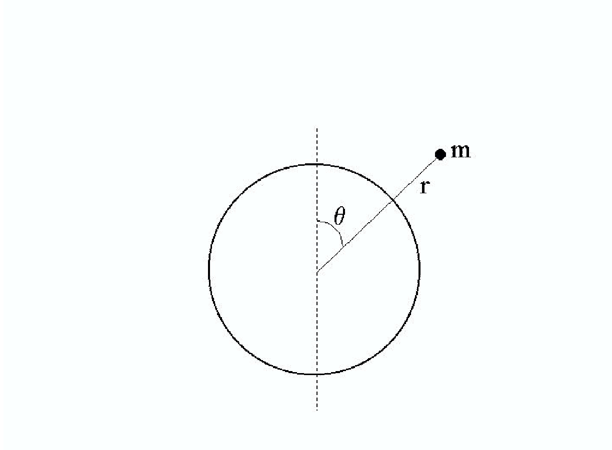

Figure 3A.

Problem MC2.

A particle starts its motion infinitely far away from the center of the central field $U(r) = -A/r^n$ ($n>2$, $A>0$). The initial speed of the particle is v_o. Determine the effective cross-section for the particle to fall to the center of the field.

Problem MD1.
Consider a system with two degrees of freedom whose Lagrangian is
$L=\frac{1}{2}[(dx/dt)^2+(dy/dt)^2]-\frac{1}{2}\omega_o^2(x^2+y^2)+\alpha xy$ (two identical one-dimensional harmonic oscillators of the same frequency ω_o coupled by an interaction $-\alpha xy$).
(a) Find the frequencies and the normal coordinates for small oscillations.
(b) Describe the oscillations of x and y for $\alpha \ll \omega_o^2$ (weak coupling). You should get what is referred to as "beats". Find the frequency of the beats.

Problem MD2.
Determine the amplitude for the oscillations of a one-dimensional harmonic oscillator of mass m and eigenfrequency ω under a force which is zero for $t<0$, $F_o t/T$ for $0<t<T$, and F_o for $t>T$, if up to time $t=0$ the system is at rest in equilibrium. Describe what happens to this amplitude if the force is applied very slowly (i.e. T is very large) – the adiabatic application of the force.
Possibly useful hints. (1) To solve the equation of motion for $0<t<T$, recall that the solution is given by the general solution of the corresponding homogeneous equation (that with no external force) plus any particular integral of the inhomogeneous (full) equation. The particular integral is not too hard to guess (try some simple functions that you know). (2) To get the equation of motion for $t>T$, ask yourself a question: what is the difference between the motion of a free oscillator and the motion of the same oscillator to which a constant force is applied.