Qualifying Examination
Electricity and Magnetism
January 12, 2006

PROBLEM EA.
There is an infinitely long cylinder along the z-axis of radius R, with a charge λ per unit length, which is radially distributed as $\rho(r) = Ar$, where $r = \sqrt{x^2 + y^2}$.

a.) Find the electric field and potential inside and outside the cylinder.

b.) Sketch the potential $\Phi(r)$ as function of r, if $\Phi(R) = 0$.

PROBLEM EB.
A 1 GeV (10^9 eV) photon collides head-on with a 1 meV (10^{-3} eV) photon (cosmic backgound photon) in the laboratory frame.

a.) What is the velocity of their center of mass system relative to the laboratory frame (direction of $\vec{\beta}$ and the numerical value of $1/\sqrt{1-\beta^2}$)?

b.) What are their respective energies in the center of mass frame?

PROBLEM EC1.
A magnetic dipole, with dipole moment $\vec{m}$ is located on the z-axis at $z = d$ in front of a magnetic medium with $\mu = \infty$. The medium fills the space $z \leq 0$ (i.e. the flat interface of the medium is the $x - y$ plane at $z = 0$). The orientation of $\vec{m}$ is along the z-direction.

a.) First consider a free magnetic dipole (at the origin). Its vector potential is $\vec{A} = \frac{\vec{m}\times\vec{r}}{r^3}$ or, in spherical coordinates, $\vec{A} = (A_r, A_\theta, A_\phi)$ has $A_r = 0$, $A_\theta = 0$, $A_\phi = \frac{m \sin\theta}{r^2}$. What is the $\vec{B}$ field in cartesian coordinates? What is the $\vec{B}$ field in spherical coordinates?

b.) What is the $\vec{B}$ field for $z \geq 0$ if the magnetic dipole is in front of a magnetic medium as described above. (Use the method of images to satisfy the boundary conditions at the interface).

c.) Find the force exerted on the dipole by the medium.

PROBLEM EC2.
An electric dipole, with magnitude oscillating in time, is placed parallel to a grounded, conducting plate at a distance d. Take the wavelength $\lambda \gg d$.

a.) Find the radiation pattern $\frac{dP}{d\Omega}$, and the total radiated power P. How does this differ from a single free electric dipole?

b.) What is the radiation pattern, and the total radiated power if the dipole is oriented perpendicular to the grounded plate?

PROBLEM ED1.

Two non-conducting plates, both parallel to the $x-y$ plane, extend over the region $0 \leq x \leq a$ and $0 \leq y \leq b$. One plate is located at $z = 0$ and has a uniform charge density $-\sigma$. The second plate is located at $z = h$ and has a uniform charge σ. Assume that the distance h between the plates is much smaller than their length a, and width b, so that edge effects can be ignored. There is a uniform magnetic field $\vec{B_o} = B_o\hat{y}$.

a.) What is the Poynting vector $\vec{S}$? What is the momentum density of the electromagnetic field? What is the total momentum of the electromagnetic field?

b.) Now the magnetic field $\vec{B_o}$ is turned off in a time Δt. What is the impulse $\Delta\vec{p}$ in time Δt experienced by each plate, as derived from the induced electric field? How does this compare to the field momentum derived in part a)?

PROBLEM ED2.

A plane light wave, described by a vector potential

$$\vec{A}(x, y, z, t) = (A_x, A_y, A_z) = (0, f(x - ct), 0),$$

hits a free electron initially at rest. Here $f(x - ct)$ is a function of $x - ct$ that vanishes for $t \to -\infty$ at fixed x.

a.) Derive the equations of motion for the electron.

b.) How are the velocity components (v_x, v_y, v_z) of the electron related? (Check $v_x^2 + v_y^2$, and its time derivative.)

c.) Show that the electron is pushed into the $+x$-direction.

d.) Eliminate v_x and express v_y in terms of $f(x - ct)$.

A useful integral is $\int \frac{1}{\sqrt{c^2 - x^2}} dx = \arcsin(\frac{x}{c})$ for $x < c$.