Let $f$ be the friction force between the blocks. \[ \begin{cases} F - f = M_1 a_1, \\ f = M_2 a_2 \end{cases} \quad a_1 = a_2 \]

Thus $F = f \left(1 + \frac{M_1}{M_2}\right)$.

$F_{\text{max}} = f_{\text{max}} \left(1 + \frac{M_1}{M_2}\right)$

$f_{\text{max}} = \mu N = \mu M_1 g \quad \Rightarrow \quad F_{\text{max}} = \mu M_1 g \left(1 + \frac{M_1}{M_2}\right)$
(a) \[ \frac{dU}{dr} \bigg|_{r_{\text{min}}} = 0 \]

\[
\frac{dV}{dr} = -3 \left( -\frac{12r_0^{12}}{r^{13}} + \frac{12r_0^6}{r^7} \right) = 0
\]

\[ \Rightarrow r = r_0 \]

The depth is \[ U(r_{\text{min}}) = U(r_0) = -3 \]

(b) Approximating \( U(r) \) as quadratic potential at \( r_{\text{min}} \):

\[
\omega = \sqrt{\frac{d^2V}{dr^2}} \bigg|_{r=r_0} \cdot \frac{1}{\mu} \quad \mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_2}{2}
\]

We get \[ \frac{d^2V}{dr^2} \bigg|_{r=r_0} = \frac{12 \cdot 3 \cdot \epsilon}{r_0^2} \]

\[
\omega = 12 \sqrt{\frac{3}{m r_0^2}}
\]
For motion in central field,

\[ E = \frac{1}{2} m \dot{r}^2 + U_{\text{eff}}(r), \]

where

\[ U_{\text{eff}}(r) = U(r) + \frac{l^2}{2mr^2}, \]

and \( l \) is the conserved angular momentum of the particle.

For our case, \( l = m v_0 p \)

\[ U_{\text{eff}}(r) = \frac{m p^2 v_0^2}{2r^2} - \frac{A}{r^n} \]

The maximum value of \( U_{\text{eff}} \) is \( U_0 = \frac{1}{2} (n-2) A \left( \frac{m p^2 v_0^2}{A n} \right)^{\frac{n}{n-2}} \).

The particles which fall to the center are those for which \( U_0 < E \). The condition \( U_0 = E \) gives \( p_{\text{max}} \)

\[ \Omega = \pi \beta_{\text{max}}^2 = \pi n (n-2) \frac{2-n}{n} \left( \frac{A}{m v_0^2} \right)^{\frac{2}{n}} \]
\[ U = -\frac{GMm}{\rho} + 5.4 \times 10^{-4} \frac{GMm}{\rho} \left( \frac{Re}{\rho} \right)^2 \left( 3 \cos^2 \Theta - 1 \right) = \]

\[ -\frac{GMm}{\rho} + A \frac{3 \cos^2 \Theta - 1}{\rho^3} \quad , \quad A = 5.4 \times 10^{-4} \frac{GMm}{Re^2} \]

In polar coordinates,

\[ F_\theta = -\frac{1}{\rho} \frac{\partial U}{\partial \theta} = \frac{3A \cdot 2 \cos \Theta \cdot \sin \Theta}{\Gamma_\theta} = \frac{3A \sin 2\Theta}{\Gamma_\theta} \]

\[ F_\theta \neq 0 \quad \text{for} \quad \Theta \neq 0 \quad , \quad \Theta \neq \frac{\pi}{2} \]

For \( \Theta = 45^\circ \), \( \rho = Re \)

\[ F_\theta = \frac{3A}{Re} = 1.52 \times 10^{-3} \frac{GMm}{Re^2} \]

\[ \frac{F_\theta}{GMm/Re^2} = 1.52 \times 10^{-3} \]
The equations of motion
\[
\frac{d}{dt} \left( \frac{\partial}{\partial x} \right) - \frac{\partial}{\partial x} = 0 \\
\frac{d}{dt} \left( \frac{\partial}{\partial y} \right) - \frac{\partial}{\partial y} = 0
\]
give
\[
\begin{align*}
\dot{x} + \omega^2 x &= \omega y \\
\dot{y} + \omega^2 y &= \omega x
\end{align*}
\] (1)

Look for solution
\[
\begin{align*}
x &= A_x e^{i\omega t} \\
y &= A_y e^{i\omega t}
\end{align*}
\]

obtain from (1)

\[
\begin{align*}
A_x (\omega^2 - \omega^2) &= \alpha A_y \\
A_y (\omega^2 - \omega^2) &= \alpha A_x
\end{align*}
\] (2)

The characteristic equation
\[
\begin{vmatrix}
\omega^2 - \omega^2 & -\alpha \\
-\alpha & \omega^2 - \omega^2
\end{vmatrix} = 0
\]
gives

\[
\omega_1 = \sqrt{\omega^2 - \alpha} \\
\omega_2 = \sqrt{\omega^2 + \alpha}
\]
For \( w = w_1 \), equations (2) give \( A_x = A_y \), and for \( w = w_2 \), \( A_x = -A_y \).

Hence, \( x = \frac{Q_1 + Q_2}{\sqrt{2}} \), \( y = \frac{Q_1 - Q_2}{\sqrt{2}} \), where \( Q_{1,2} \) are the normal coordinates.

Thus, \( Q_1 = \frac{x + y}{\sqrt{2}} \), \( Q_2 = \frac{x - y}{\sqrt{2}} \).

Note: normalization coefficient \( \frac{1}{\sqrt{2}} \) can be chosen differently, answers like \( Q_1 = x + y \) etc. are acceptable.

For \( \alpha \ll w_2 \), we have \( w_{1,2} \approx w_0 + \delta \), where \( \delta = \frac{1}{2} \frac{\alpha}{w_0} \).

Thus, we have a sum of two oscillations with almost equal frequency, i.e. beats of frequency \( \delta \).

\[ w_2 - w_1 = \frac{\alpha}{w_0} \]

E.g. \( \cos(w_0 + \delta) + \cos(w_0 - \delta) = 2 \cos(w_0) \cos(\delta t) \) etc.

In general, \( x = \frac{1}{\sqrt{2}} (A e^{i\omega t} e^{i\delta t} + B e^{i\omega t} e^{-i\delta t}) = \frac{1}{\sqrt{2}} e^{i\omega t} (A e^{i\delta t} + B e^{-i\delta t}) \)

giving \( A' \cos(w_0 t + \Phi) \) (slowly varying term of freq. \( \delta \)).
For $0 < t < T$, the equation of motion is
\[ m \ddot{x} + m \omega^2 x = F(t) = \frac{F_0}{T} \]
(1)
The solution is \( x(t) = A_1 \cos \omega t + A_2 \sin \omega t + G(t) \)
where \( G(t) \) is any solution of (1).
Seek \( G(t) = B t \), gives \( B = \frac{F_0}{T m \omega^2} \).
Initial conditions then give
\[
\begin{align*}
    x(0) &= 0 \Rightarrow A_1 = 0 \\
    \dot{x}(0) &= 0 \Rightarrow A_2 = -\frac{F_0}{T m \omega^3}
\end{align*}
\]
Thus, \( x(t) = \frac{F_0}{m T \omega^3} (\omega t \sin \omega t) \) for $0 < t < T$.
(2)
For $t > T$, we have an oscillator with a constant force applied. This will simply produce a displacement of the equilibrium position.
Direct substitution into \( m \ddot{x} + m \omega^2 x = F_0 \)
gives \( \frac{F_0}{m \omega^2} \) for this displacement. Thus, for $t > T$
\[
    x(t) = C_1 \cos \omega (t - T) + C_2 \sin \omega (t - T) + \frac{F_0}{m \omega^2}
\]
(3)
Using (2) and (3), and the continuity condition of $x$ and $\dot{x}$ at $t=T$ gives:

$$c_1 = -\frac{F_0}{mT\omega^3} \sin \omega T \quad c_2 = \frac{F_0}{mT\omega^3} (1 - \cos \omega T).$$

The amplitude is

$$a = \sqrt{c_1^2 + c_2^2}.$$

$$a = \frac{2F_0}{mT\omega^3} \sin \frac{\omega T}{2}.$$  \hspace{0.5cm} e^2$$

As $t \to \infty$, $a \to 0$. In the limit of the adiabatic application of the force, the amplitude is zero ("the oscillator remains in the ground state" analogy from QM).