Rutgers - Physics Graduate Qualifying Exam
Thermodynamics & Statistical Mechanics: September 1, 2006

SA

A building at a temperature T (in K) is heated by an ideal heat pump which uses the atmosphere at T_0 (K) as heat source. The pump consumes power W and the building loses heat at a rate $\alpha(T - T_0)$. What is the equilibrium temperature of the building?

SB

A system consists of N noninteracting magnetic dipoles. Each dipole carries a magnetic moment μ which can be treated classically. If the system at a finite temperature T is in a uniform magnetic field H, find

1. the induced magnetization in the system, and
2. the heat capacity at constant H.

SC1

A Van der Waal's gas has the equation of state

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

1. Discuss the physical origin of the parameters a and b. Why is the correction to p invesely proportional to V^2?
2. The gas undergoes an isothermal expansion from volume V_1 to volume V_2. Calculate the change in the Helmholtz free energy.
3. From the information given can you calculate the change in internal energy? Discuss your answer.

SC2

Consider a photon gas enclosed in a volume V and in equilibrium at temperature T. The photon is a massless particle, so that $\varepsilon = pc$.

1. What is the chemical potential of the gas? Explain.
2. Determine the parametric dependence of the number of photons in the volume on the temperature. (You do not need to evaluate the numerical prefactor.)
3. One may write the energy density in the form

   $$\frac{\bar{E}}{V} = \int_0^\infty \rho(\omega) d\omega.$$

   Determine $\rho(\omega)$, the spectral density of the energy.
4. Determine the parametric dependence of the energy, $\bar{E}$, on the temperature.
   (You do not need to evaluate the numerical prefactor.)

SD1

Suppose a new kind of particle is discovered. This particle is known as the weirdon since it obeys weird statistics in which a given state may contain 0, 1, or 2 particles. Furthermore, weirdons are one dimensional and we will be considering a gas of non-interacting weirdons confined to a straight line of length L. The weirdons are weakly coupled to a thermal reservoir at a temperature, T, and the weirdon mass is m.

1. Suppose the chemical potential of the weirdons is μ.
   What is the occupancy of a state with energy ϵ?
   In addition, give numerical values of the occupancy for
   $(\mu - \epsilon)/\tau = -\infty$,
   $(\mu - \epsilon)/\tau = 0$, and
   $(\mu - \epsilon)/\tau = +\infty$,
   where $\tau = k_B T$, and k_B is the Boltzmann constant.

2. Suppose the weirdon gas is cold ($\tau \to 0$) and contains N weirdons.
   What is the chemical potential?
   What is the total energy of the weirdon gas?
   Be sure to eliminate μ from your expression for the energy.

3. The low temperature heat capacity and entropy of the weirdon gas are proportional to the temperature to some powers, $C \propto \tau^{\alpha}$, $S \propto \tau^{\beta}$.
   What are α and β?

SD2

Consider a 3D gas of electrons in a large, cubical box of size L, with periodic boundary conditions under a uniform magnetic field B in the vertical direction. In this problem we will ignore the spin of the electrons.

1. What is the degeneracy of the quantized energy levels? You do not need to worry about the derivation as long as you are able to provide the correct answer.

2. Calculate the grand partition function.

3. Determine the magnetic susceptibility per unit volume in the high temperature limit in terms of electron number density, n, the temperature, T, and universal constants.