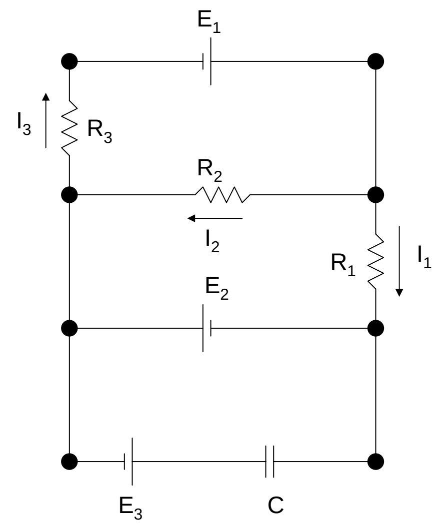
Rutgers - Physics Graduate Qualifying Exam
Electricity & Magnetism: August 30, 2006

EA

In the circuit shown in the figure below, $R_1 = 5\,\Omega$, $R_2 = 3\,\Omega$, $R_3 = 5\,\Omega$, $E_1 = 4$ V, $E_2 = 8$ V, $E_3 = 3$ V, and $C = 6\mu F$. Under steady state conditions, find the currents (magnitude and direction) I_1, I_2, I_3, and the charge on the capacitor. Give both the symbolic answers and the numerical values. For the currents, you can leave your symbolic answers as long strings - you do not have to tidy them up.



EB

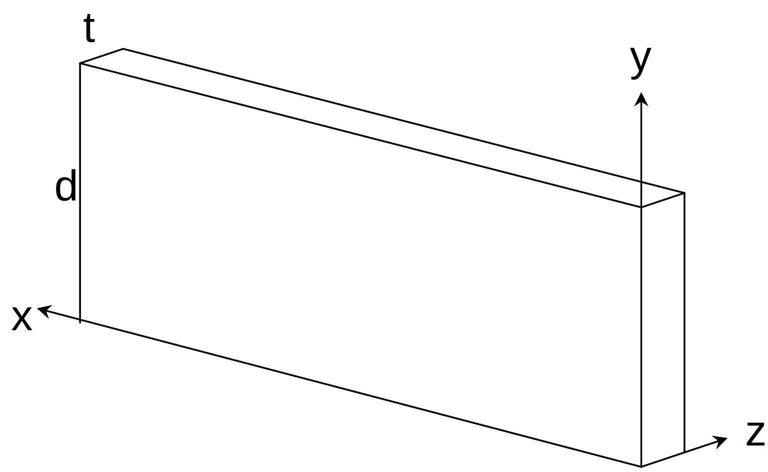
The ultimate goal of this problem is to determine the capacitance of a straight telephone wire suspended at a fixed height h above flat ground surface. The length of the wire is l, the radius of the wire is a, and the ground potential is, of course, equal to zero.

1. As a first step, determine the capacitance of a system consisting of two parallel straight wires. The length of each wire is l, the radius of each wire is a, and the distance between the wires is $2h$. Assume that $l \gg h \gg a$. You can assume that the charges on the upper and lower wires are q and $-q$, respectively.
   Hint: it might be useful to recall the expression of a potential produced by a long straight wire.

2. Using the result of part (a) and symmetry arguments, find the capacitance of the telephone wire described in the beginning of the problem.
   Hint: Consider the electric potential at an imaginary plane located in between the wires of part (a). Specifically, consider the plane that is parallel to the wires, perpendicular to the shortest line connecting the wires, and cutting the latter line in half.

EC1

1. A slab of metal shown in the figure below has width d and thickness t. Constant current I is flowing along the x direction in the slab. A uniform magnetic field B is applied in the z direction. The charge carrier concentration in the metal is n, and each carrier has charge q. Find the Hall voltage. That is, find the potential difference in the y direction across the sample.



2. Now imagine that instead of the metal slab, you have a blood vessel of the same cross section (width d, thickness t). The blood flows in the x direction. The blood contains charged ions of an unknown nature, and the ions move together with the blood flow. Magnetic field B is applied to the blood vessel in the z direction, and the "Hall" voltage V across the vessel in the y direction is measured. Find the speed of the blood in the vessel.

EC2

1. Consider a charged particle moving in a uniform magnetic field B perpendicular to the field's direction. The magnitude of the field is changing extremely slowly as a function of time. Show that under such conditions, $v^2/B =$const. Here v is the speed of the particle. Note that v^2/B will remain constant only if the magnetic field is varying very slowly, that is, v^2/B is what is called an adiabatic invariant. It is easy to show (you do not have to do this) that if the particle also has a component of the velocity along the field, then $v_{perp}^2/B =$const, where v_{perp} is the component of the velocity perpendicular to the magnetic field.

2. Now consider a magnetic field that is pointing along the x axis, and whose magnitude slowly increases with x. A charged particle with a drift velocity along the positive direction of the x axis is moving in the field. (The drift velocity is the velocity of the center of the circle along which the particle is moving in the $y - z$ plane). At the point where the magnitude of the magnetic field is B_0, the velocity of the particle makes angle α with the magnetic field. Find the approximate value of the magnetic field where the particle "gets reflected by the region of high magnetic field", i.e. where the drift velocity changes from positive to negative. You can assume that the adiabatic invariant of part (a) holds under the conditions of the problem.

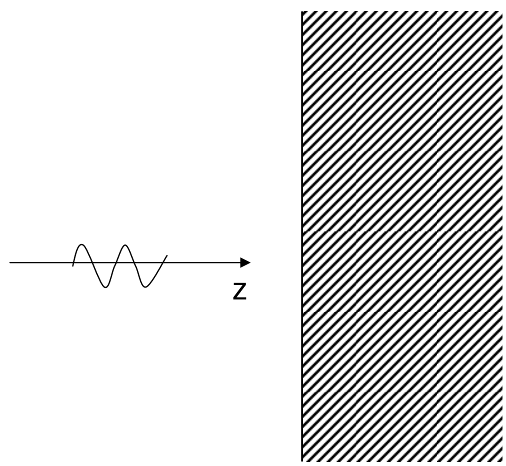
ED1

Plane electromagnetic wave of angular frequency ω is propagating in vacuum in the z direction. It hits a very large flat piece of metal whose surface is in the $x-y$ plane, as shown in the figure. The metal has conductivity λ and magnetic permeability μ. Show that the electric field of the electromagnetic wave decays exponentially inside the metal. Find the characteristic attenuation length. In your calculations, neglect the displacement current (it is always much smaller than the conduction current in metals). To check your answer, consider a radio wave of frequency $f = 10^5$ Hz falling on copper. You should get the penetration depth of the order of 10^{-4} m.

Use $\lambda = 5.9 \times 10^7\ \Omega^{-1} m^{-1}$, $\mu = 1$, $\mu_0 = 4\pi \times 10^{-7}$ if you are using the SI units.

Or use $\lambda = 5.3 \times 10^{17}$, $\mu = 1$ if you are using Gaussian units.

The following vector identity might be useful for solving this problem. For any vector $\vec{a}$, $\nabla \times \nabla \times \vec{a} = \nabla(\nabla \cdot a) - \nabla^2 \vec{a}$.



ED2

Two electric charges, e_1 and e_2, are moving with the same velocity $\vec{V}$. Let $\vec{R}$ be the radius vector from e_2 to e_1, and let θ be the angle between $\vec{R}$ and $\vec{V}$. The speed $|\vec{V}|$ is **not** small compared to the speed of light. Find the force between the two charges in this coordinate system. Give all the components of the force vector. Assume that the charges are moving in the xy-plane, and that $\hat{x}$ is the direction of motion.