Rutgers - Physics Graduate Qualifying Exam
Classical Mechanics: August 30, 2006

MA

Figure 1 shows three monkeys hanging from massless ropes, that hang from massless, frictionless pulleys. Monkey A has mass $2m$, while monkeys B and C have mass m each.

1. Monkey A climbs up towards a banana (not shown), climbing a distance d along the rope. Give the change in the vertical position of each of the three monkeys.
2. Monkeys B and C climb up towards a banana (not shown), each climbing a distance d along the rope. Give the change in the vertical position of each of the three monkeys.
3. Monkey C climbs up a distance d along the rope it holds. Give the change in the vertical position of each of the three monkeys.

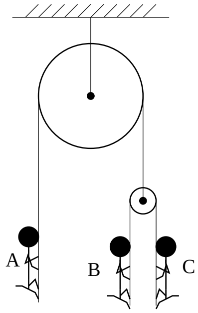

Figure 1: The three monkeys of problem MA.

MB

Figure 2 shows a block of mass M lies on a plane inclined at an angle θ to horizontal. It is attached through a massless rope and massless, frictionless pulley to a block of mass m, that hangs against a vertical side. The coefficient of friction between either block and the planes they touch is μ.

1. Under what conditions do the blocks not move?
2. The entire system is now accelerated to the right, at acceleration a.

Under what conditions do the blocks not move?

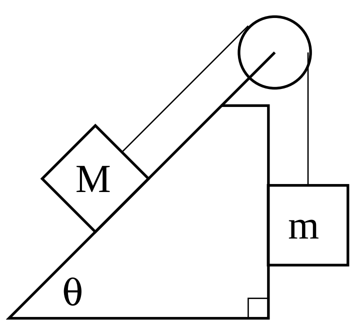

Figure 2: The blocks, planes, and pulley system of problem MB.

MC1
On top of a frictionless table we have a massless spring with constant k fixed to the table at one end. At the other end of the spring, a mass m is attached.

1. The spring is stretched slightly and released. What is the frequency ω of small oscillations?
2. The spring may rotate freely about its fixed end. The mass is pushed in the $\hat{\theta}$ direction, so it is given an angular momentum L_z. By what amount is the spring stretched?
   Write your answer in terms of L_z and the radial position of the rotating mass.
3. The revolving mass is then given a small tap in the radial direction. What is the frequency ω of small oscillations?

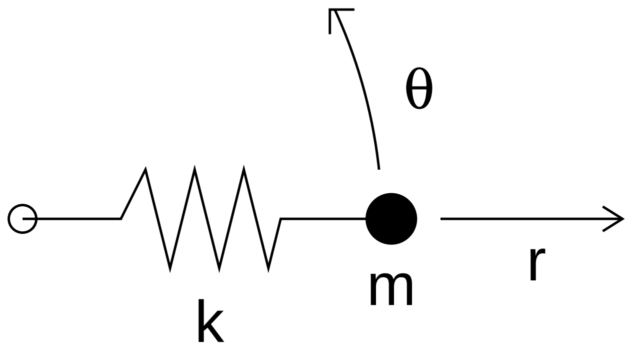

Figure 3: The spring and attached mass of problem MC1.

MC2
A pendulum consists of a massless rigid rod of length l with small mass m at the bottom end.

1. What is the frequency ω of small oscillations?
2. The pendulum is free to move to any orientation, it does not move simply in a plane. The pendulum is rotated with an azimuthal angular velocity $\dot{\phi}$. What is the equilibrium angle θ of the pendulum relative to vertical as a function of the angular momentum, L_z?
3. At time t=0, your professor gives the pendulum a small push, in the $\hat{\theta}$ direction. Does the pendulum undergo small oscillations? If so, give the frequency ω.

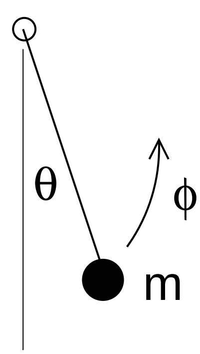

Figure 4: The pendulum of problem MC2.

MD1

Shown in Figure. 5 are two small blocks of mass m attached to a larger block of mass M through springs of constant k. All surfaces are frictionless. The position of the large block is given by x, while the smaller block positions, relative to the larger block, are given by x_1 and x_2, with, e.g., $F_1 = -kx_1$.

1. How many normal modes are there? Describe them qualitatively.
2. Work out the oscillation frequencies for the normal modes.
3. Write down a general solution for the motions of this system, when there are no external forces (other then gravity).

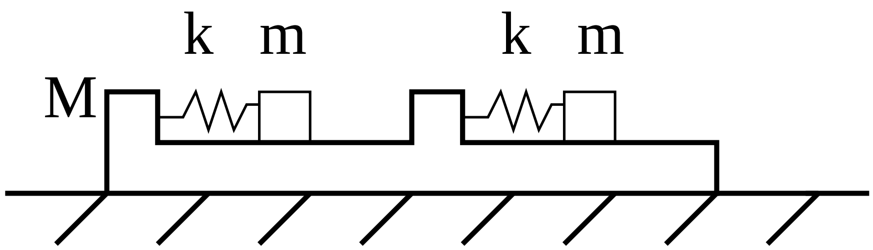

Figure 5: The blocks of problem MD1.

MD2

A large ring of mass M hangs from the ceiling by a rope. Two small rings of mass m are mounted on the larger ring, so that they start from the top of the ring and slide down opposite sides of it, as shown in Fig. 6.

1. For what values of the ratio m/M will the large ring rise, as the small rings descend around it?
2. For $M = 0$, at what angle does the large ring start to rise?

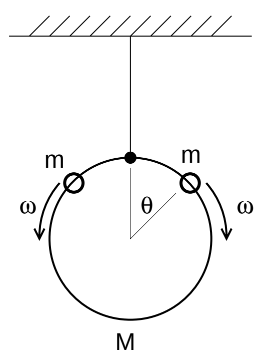

Figure 6: The rings of problem MD2.