where the sum is over occupied proton states. For each state we have
\[
\langle \psi_n | z^2 | \psi_n \rangle = \frac{(1/2)m \omega_z^2 \langle z^2 \rangle}{(1/2)m \omega_z^2} = \frac{\langle V_z \rangle}{(1/2)m \omega_z^2} = \frac{(1/2)(n_z + 1/2) \hbar \omega_z}{(1/2)m \omega_z^2} = \left( n_z + \frac{1}{2} \right) \frac{\hbar}{\omega_z},
\]
where the virial theorem has been used to show \( \langle V_z \rangle = E/2 \) for the harmonic oscillator potential. Thus, for a specific state,
\[
2\langle z^2 \rangle - \langle x^2 \rangle - \langle y^2 \rangle = 2 \left( n_z + \frac{1}{2} \right) \frac{\hbar}{\omega_z} - \left( n_x + \frac{1}{2} \right) \frac{\hbar}{2\omega_z} - \left( n_y + \frac{1}{2} \right) \frac{\hbar}{2\omega_z} = \left( 2n_z - \frac{1}{2} n_x - \frac{1}{2} n_y + \frac{1}{2} \right) \frac{\hbar}{\omega_z}.
\]
The two protons in the first shell contribute \((1/2)\hbar/\omega_z\) each, and the two protons in the second shell contribute \((5/2)\hbar/\omega_z\) each. The expectation of the quadrupole operator is thus \(6\hbar/\omega_z\) when all contributions are summed.

c) (2 points) The potential is not spherically symmetric, so \([L^2, V] \neq 0\), and the energy eigenstates are not eigenstates of \(L^2\). However, the potential is invariant under rotations about the z axis. Thus \([L_z, V] = 0\), and the energy eigenstates are eigenstates of \(L_z\).