“Dark Energy” Models

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Physics 690 – Prof. S. Jha

- An accelerating universe at first glance suggests a cosmological constant.
- But as we discussed earlier, motivating such a constant from theory is difficult.
- Perhaps rather than new matter sources, we should consider modifying gravity.
- HET-inspired models often feature compact extra dimensions, but we need modifications at large scales ($\sim cH^{-1}$).

Accelerated Universe from Gravity Leaking to Extra Dimensions

PRD 65, 044023 (2002)

C. Deffayet, G. Dvali, G. Gabadadze

A Brane in a Bulk

Suppose we live in a 4D brane embedded in an infinite 5D bulk. We could write our gravitational action:

$$S = \frac{M_{(5)}^3}{2} \int d^5X \sqrt{|\tilde{g}|}\tilde{R} + \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{|g|} R,$$

The Planck masses set the scales for the two terms. The Ricci scalar *R* contains information on the curvature of space. Varying it w.r.t. the metric yields Einstein's equation.

Non-relativistic potential

Even non-relativistically, the extra dimension results in a gravitational potential which goes like *1/r* locally but like *1/r*^{2} at large distances, with the transition occurring at

$$r_c = \frac{M_{\rm Pl}^2}{2M_{(5)}^3}$$

Cosmological dynamics

The 5D metric reduces to the familiar 4D FRW metric at the location of the brane.
However, the dynamics are different.

$$H^2 + \frac{k}{a^2} = \left(\sqrt{\frac{\rho}{3M_{\mathrm{Pl}}^2} + \frac{1}{4r_c^2}} \pm \frac{1}{2r_c} \right)^2$$

At early times, matter density dominates and we recover ordinary cosmology, thus making the theory compatible with Big Bang nucleosynthesis, etc.

Cosmological dynamics

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Over time, the matter dilutes. Choosing the + sign and no matter, the modified Friedmann equation reduces to:

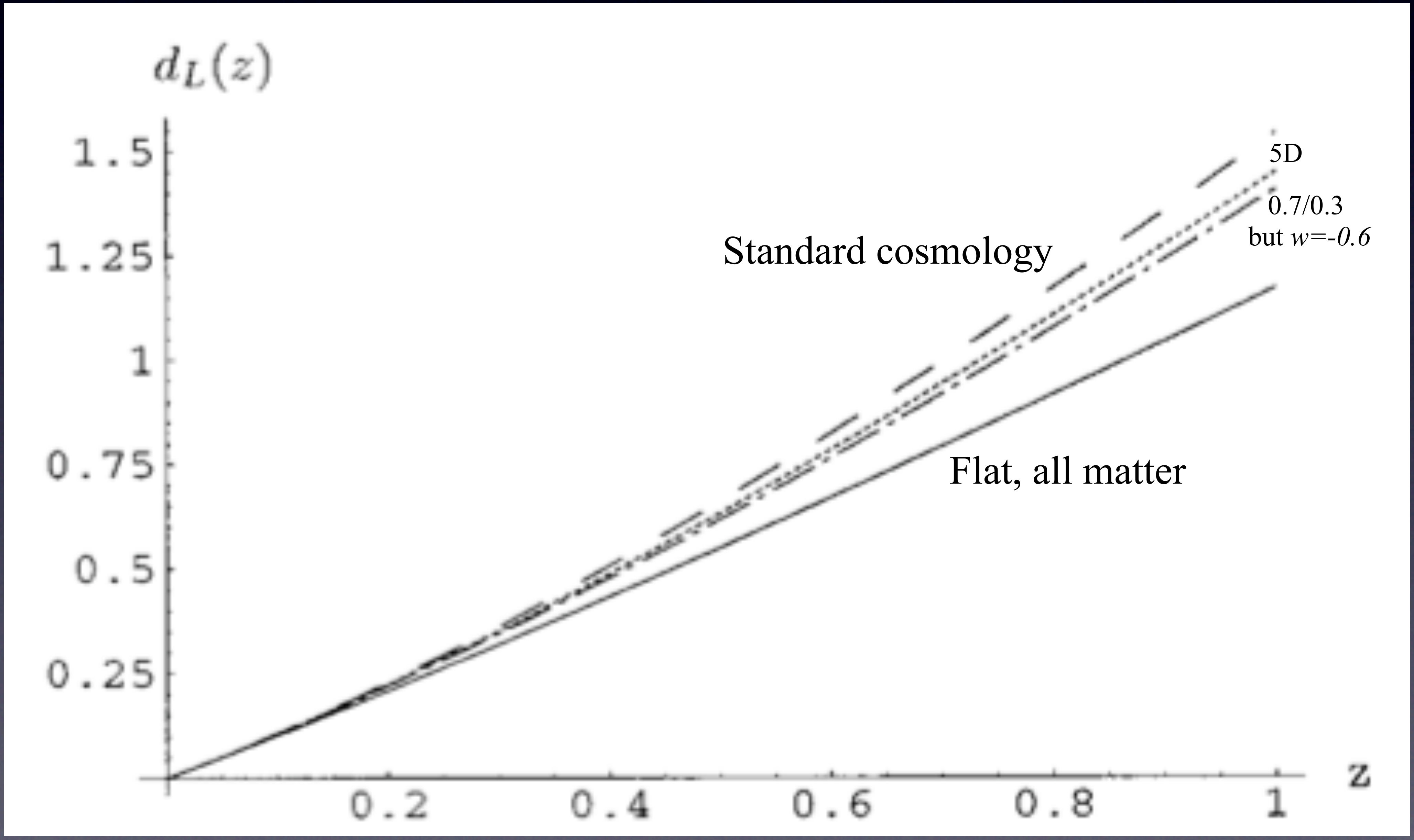
$$H^2 + \frac{k}{a^2} = \frac{1}{r_c^2}$$

The crossover radius is behaving like a source term!

Self-inflation

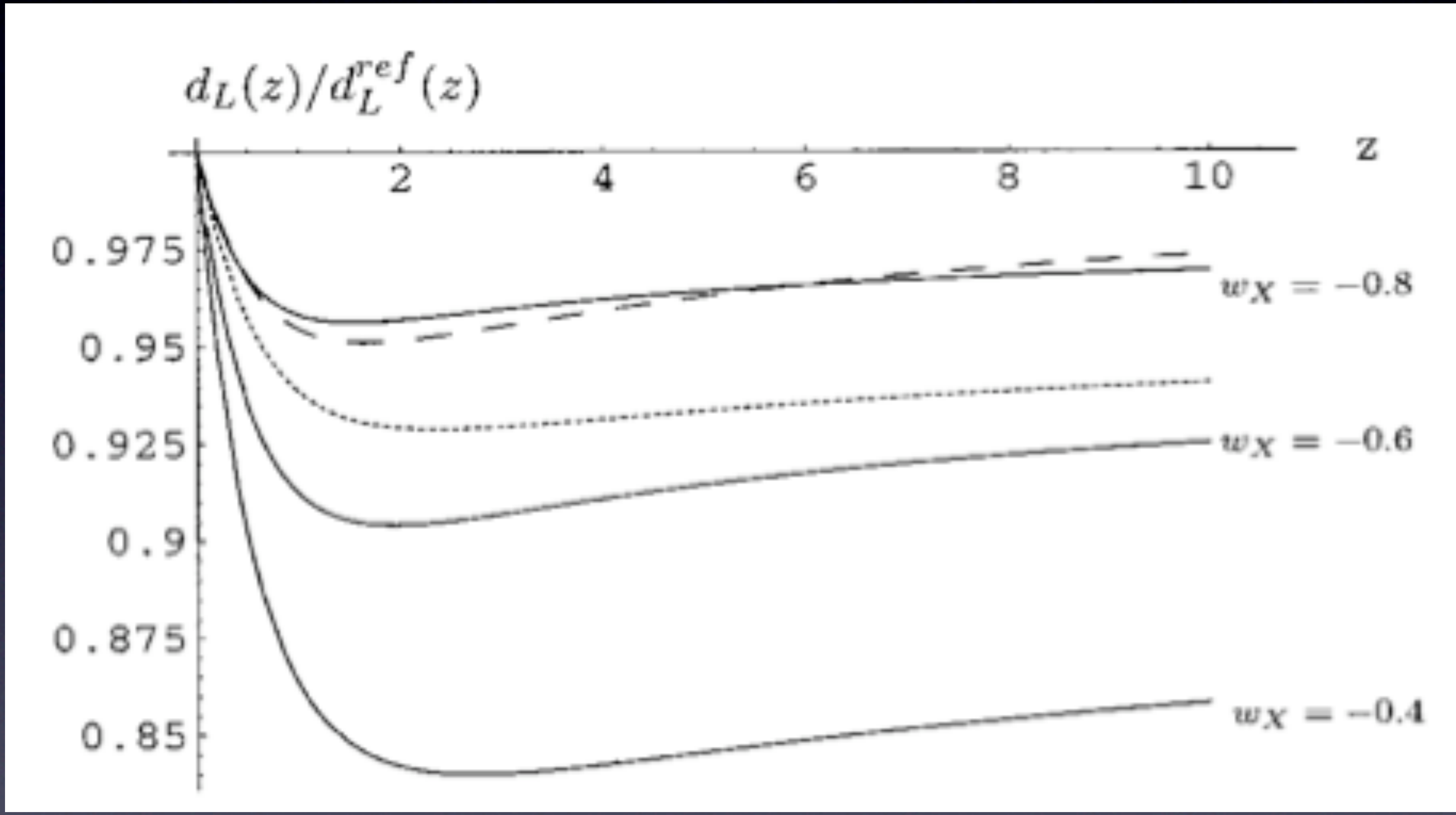
Supernovae tests

From our modified 5D metric we can calculate distance measures as functions of redshift (*cf.* first week's paper).



Comparison with DE

Compare luminosity difference in various scenarios vs. standard 0.7/0.3, $w = -1$ cosmology.

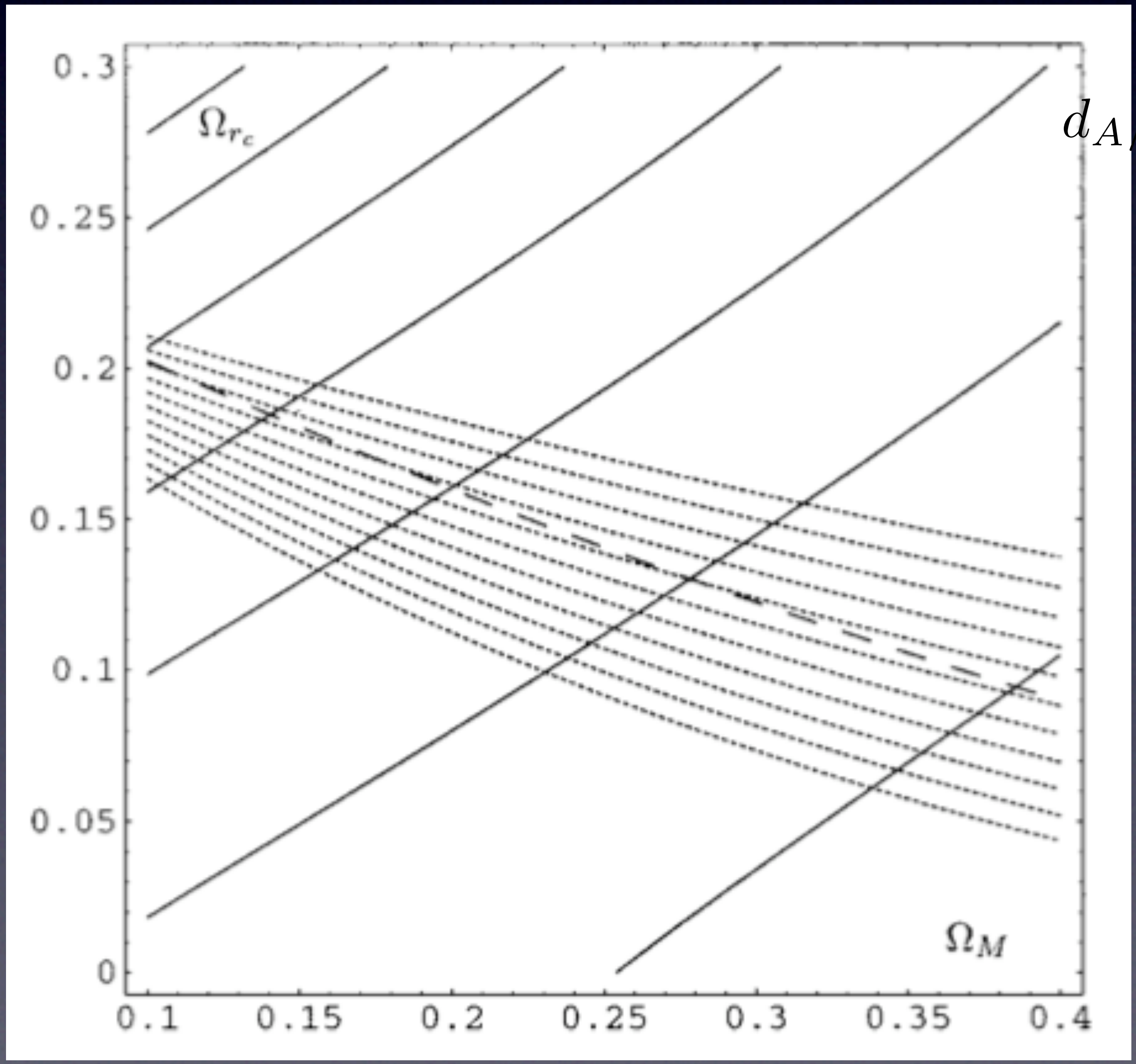


The extra dimension causes acceleration effects not compatible with a pure cosmological constant which could in principle be detected.

CMB constraints

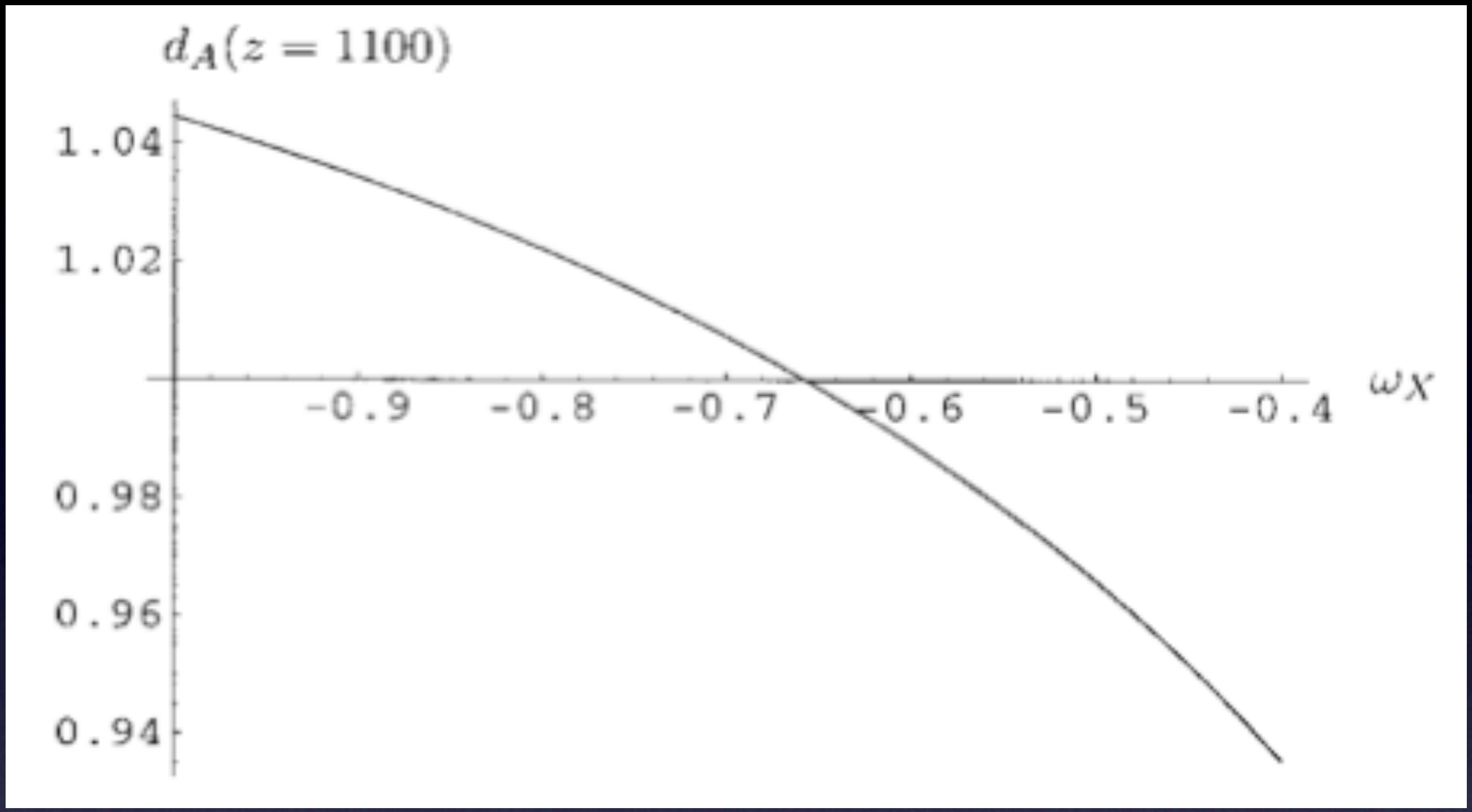
Luminosity distances are actually degenerate between (Ω_M, Ω_Λ).

The location of the first acoustic peak in the CMB can lift the degeneracy.



$d_A/d_A^{(5D)}$

CMB constraints



Ratio angular diameter distance at last scattering (*z=1100*) in standard cosmology to that in the 5D theory, assuming a flat universe and Ω_M=0.3.

First peak is displaced to lower multipoles in the 5D theory.

But how about just modifying the Lagrangian?

Higher order terms in *R* are suppressed like M_{Pl}, and so are unimportant on cosmological scales.

Carroll *et al.* examine *lower order* in *R* terms.

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\left(R - \frac{\mu^4}{R}\right)$$

Cosmology of Generalized Modified Gravity Models

PRD 71, 063513 (2005)
S. Carroll, *et al.*

A choice of frames

With a modified Lagrangian, we will obviously obtain different gravitational field equations. On the other hand, a conformal transformation allows us to recast the problem so that the gravitational field equation maintains its familiar form at the expense of introducing an auxiliary scalar "matter" field.

It is well known that scalar fields in GR can play the role of a cosmological constant.

3 possible outcomes

The fate of a vacuum will depend on the initial conditions of the scalar field.

If conditions are such that the field just reaches the maximum of its potential, yielding a constant acceleration. (unstable)

If the field overshoots the maximum, there will be a non-constant acceleration ($w = -2/3$).

If the field can't make it over the peak, it rolls back to zero and the universe will recollapse.

These conclusions also hold generally in the presence of matter.

In order to control when expansion effects become important, we must adjust μ. So some fine tuning seems necessary to get it to start at the present epoch.

But general late-time expansions can be accommodated by an appropriate R^{-n} term in the action.

$$w_{\mathrm{eff}} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}.$$

The latest WMAP results set a limit $-0.89 < w < -1.14$.

For the generalized models we are considering, the Friedmann equation is a 3rd order differential equation. We want to look for solutions which result in an asymptotic expansion so that the scale factor $a(t) \propto t^p$. It is easy to see for such a solution

$$\dot{H} = -H^2/p$$

If we define $v = -H^2/\dot{H}$

then we want solutions that provide $v \rightarrow$ some constant p.

In particular $p > 1$ for an accelerating expansion.

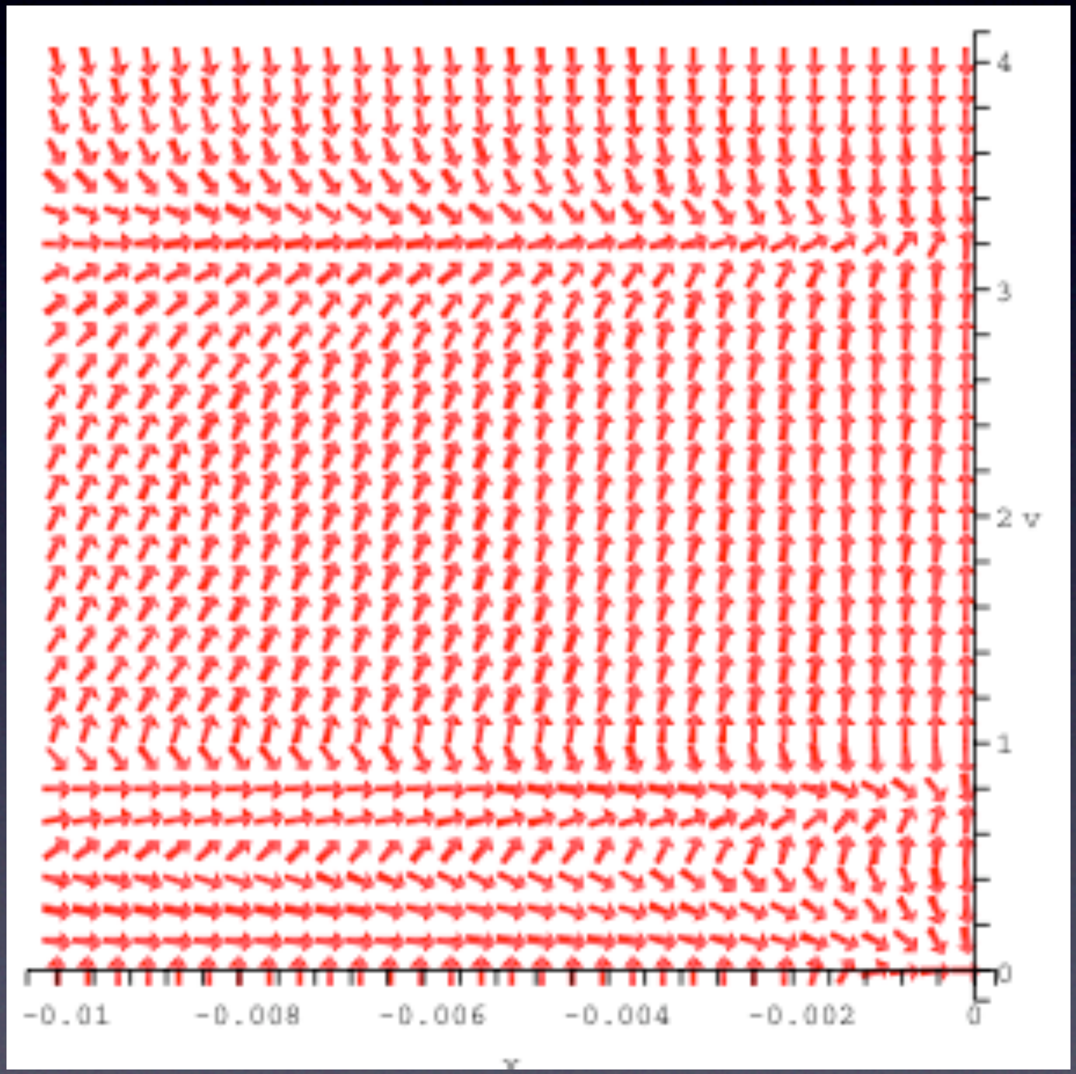
We are considering the addition of lower order terms to the action, which contains scalars built from the curvature tensor. Besides the usual $R = R^{\mu}_{\mu}$, some novel choices are:

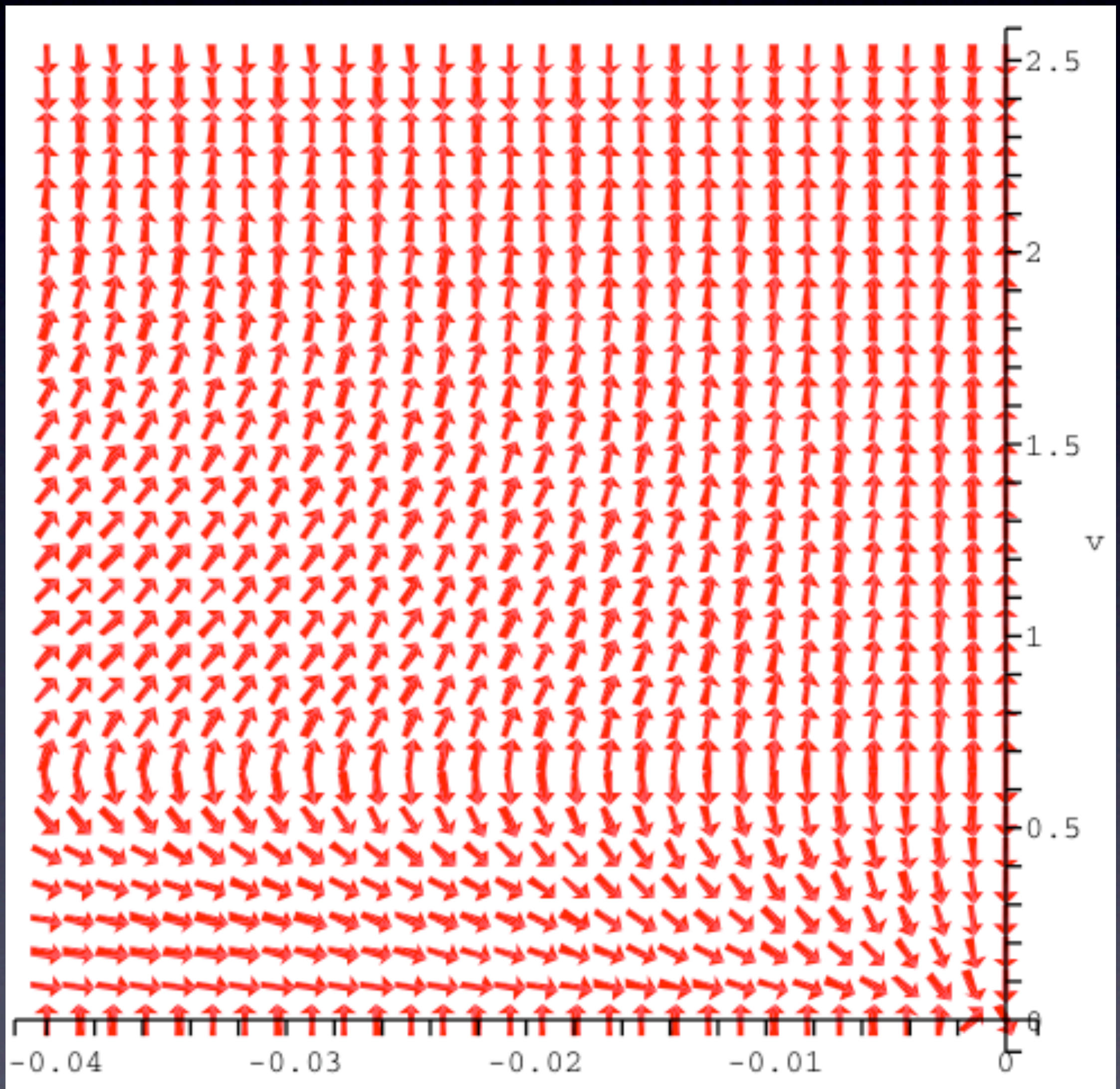
$$P \equiv R_{\mu\nu} R^{\mu\nu} \qquad Q \equiv R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho}$$

This paper specializes to terms of the form

$$f(R, P, Q) = -\frac{\mu^{4n+2}}{(aR^2 + bP + cQ)^n},$$

Various sample terms are considered.
Look for constant curvature (maximally symmetric) solutions to the Friedmann equation.
Graphical phase plots show singularities and attractors.





Theories involving inverse powers of P, Q.

Conclusion

A rich variety of behaviors are possible, even without extra dimensions or vacuum energies, arising from reasonable lower order terms in the gravitational action.

The small value of μ ensures that these effects only show up at the appropriate times, although motivating the existence of the extra terms and the value of mu remains an outstanding problem.