Masses of Nebulae and Clusters of Nebulae

Fritz Zwicky - 1937
Fig. 1.—Velocity of rotation in nebulae
Virial Theorem For Nebulae
\[ \sum F = ma = \frac{m v^2}{r} = \frac{G M m}{r^2} \]

\[ v = \sqrt{\frac{G M}{r}} \]

\[ U[r] = -\frac{G m M}{r} \]

\[ T = \frac{m v^2}{2 r} = \frac{G M m}{2 r} = -\frac{U[r]}{2} \]
Deriving The Mass...

$$\overline{v^2} = 3 \overline{v_s^2}.$$ 

$$\mathcal{M} > \frac{3R \overline{v_s^2}}{5 \Gamma}.$$ 

From the observations of the Coma cluster so far available we have, approximately,$^5$

$$\overline{v_s^2} = 5 \times 10^{15} \text{cm}^2 \text{sec}^{-2}.$$ 

$$\mathcal{M} > 9 \times 10^{46} \text{gr}.$$ 

$$\overline{M} > 9 \times 10^{43} \text{gr} = 4.5 \times 10^{10} M_\odot.$$ 

$$\gamma = 500$$
What do we expect the mass to light ratio to be???
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- Uh...1-3.
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- What could this non-luminous material possibly be???
Dark Matter!
X-ray measurements on the mass of M-87

Written by D. Fabricant, M. Lecar and P. Gorenstein.
Flux of 0.7 – 3.0 keV X-Rays
Temperature Profile of M-87

Fig. 3.—The temperature profile of M87 derived by fits to the spectral data. $N_H$ has been fixed at $2.5 \times 10^{20}$ and the ratio of iron
Electron Density

[Graph showing electron density as a function of radial distance (arcmin)].

[Inset graph showing logarithmic relationship between radial distance (arcmin) and another variable (log p/log r)].
Equation of State for M87

\[
\frac{dP_{\text{gas}}}{dr} = -\frac{G M_*(r) \rho_{\text{gas}}}{r^2}
\]

\[
P_{\text{gas}} = \frac{\rho_{\text{gas}} K T_{\text{gas}}}{\mu M_H}
\]

\[
- \frac{K T_{\text{gas}}}{G \mu M_H} \left( \frac{d \log \rho_{\text{gas}}}{d \log r} + \frac{d \log T_{\text{gas}}}{d \log r} \right) r = M_*(r)
\]

If \( T_{\text{gas}} \) is a constant, this simplifies to:

\[
- \frac{K T_{\text{gas}}}{G \mu M_H} \left( \frac{d \log \rho_{\text{gas}}}{d \log r} \right) r = M_*(r).
\]
Results!

Graphs showing the relationship of $M_{\text{halo}}(r)$ with radial distance.
Conclusions

- Halo mass lies within a range of $1.7 \times 10^{13} - 4 \times 10^{13}$ Solar masses.
- Halo extends to a minimum extent of 50’ to 60’.
- Dark matter comprises over 99% of M87’s mass!