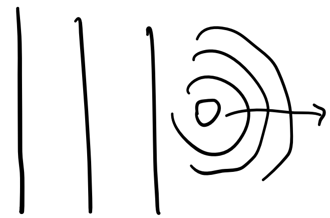
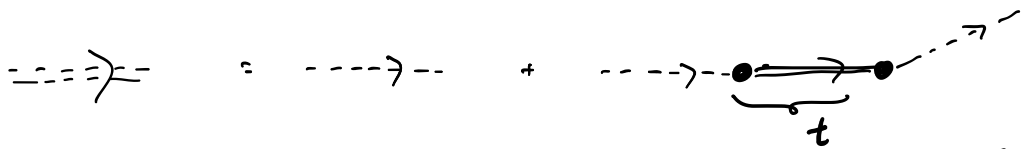
Remarks about G and phaseshift

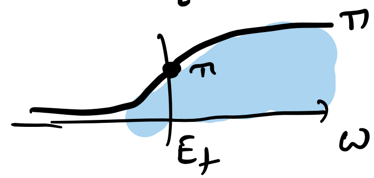
$$G_f(\omega - i\eta) = \frac{1}{\omega - (E_f + i\Delta)}$$ Advanced G.f

$$A_f(\omega) = \frac{1}{\pi}\,\mathrm{Im}\, G_f(\omega - i\eta) = \frac{\Delta}{(\omega - E_f)^2 + \Delta^2}$$ Resonant level.





$$-G^{-1}(\omega - i\eta) = |G^{-1}|\, e^{i\delta}$$



$$\delta = \tan^{-1}\left(\frac{\Delta}{E_f - \omega}\right)$$

$$\left(\frac{e^{-ikr}}{r} - \frac{e^{ikr}e^{2i\delta}}{r}\right) = \frac{e^{i\delta}}{r}\, 2i \sin(kr - \delta)$$

$$\propto \sin(kr - \delta)$$

$$t(\omega) = V^2 G_f(\omega + i\eta) \sim e^{i\delta}\sin\delta$$

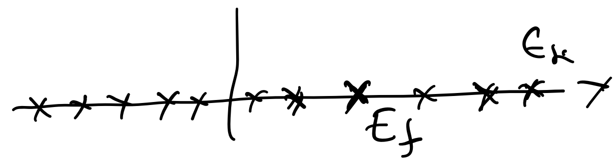
$$S = 1 - 2\pi i \rho\, t(\omega + i\eta) = e^{2i\delta}$$

$$\Rightarrow \quad t = \left(\frac{S - 1}{-2\pi i \rho}\right) = -\frac{(e^{2i\delta} - 1)}{2\pi i \rho} = -\frac{e^{i\delta}\sin\delta}{\pi\rho} = -\frac{\sin\delta}{\pi\rho(\cos\delta - i\sin\delta)} = -\frac{1}{\pi\rho(\cot\delta - i)}$$

$$S = 1 - \frac{2\pi i \rho V^2}{\omega - E + i\Delta} = \frac{\omega - E - i\Delta}{\omega - E + i\Delta} = \frac{G^{-1}(\omega - i\delta)}{G^{-1}(\omega + i\delta)}$$

$$V^2 G_f = \frac{1}{\pi\rho}\,\frac{\Delta}{(\omega - E_f) + i\Delta} = \frac{1}{\pi\rho}\,\frac{1}{i + \left(\frac{\omega - E_f}{\Delta}\right)} = -\frac{1}{\pi\rho}\,\frac{1}{\left[\left(\frac{E_f - \omega}{\Delta}\right) - i\right]}$$

$$0 = \mathrm{Det}\left[\left(\begin{array}{c|c} \omega - E_f & -V^* \\ \hline V & \omega-\epsilon_k\end{array}\right)\right] = \mathrm{Det}\left(\begin{array}{c|c} \omega - E - \sum \frac{V^*V}{\omega-\epsilon_k} & 0 \\ \hline V & \omega-\epsilon_k\end{array}\right)$$



$$(\partial_\tau + H) = (H - i\omega_n) = -\mathcal{G}^{-1}$$

$$\mathrm{Det}\begin{bmatrix}A&B\\C&D\end{bmatrix} = \mathrm{Det}\left(\begin{bmatrix}1&-BD^{-1}\\0&1\end{bmatrix}\begin{bmatrix}A&B\\C&D\end{bmatrix}\right) = \mathrm{Det}\left(\begin{array}{c|c}A-BD^{-1}C & 0\\ \hline C & D\end{array}\right)$$

$$= \mathrm{Det}(A-BD^{-1}C)\,\mathrm{Det}(D)$$

$$\int \mathcal{D}[\bar\alpha,\alpha]\int\mathcal{D}[\bar\beta,\beta]\exp\left[(\bar\alpha,\bar\beta)\begin{pmatrix}A&B\\C&D\end{pmatrix}\begin{pmatrix}\alpha\\\beta\end{pmatrix}\right] = \int\mathcal{D}[\bar\alpha,\alpha]\,e^{\bar\alpha A\alpha}\times \underbrace{\int\mathcal{D}[\bar\beta,\beta]\,e^{[\bar\beta D\beta + \bar J\beta + \bar\beta j]}}_{\mathrm{Det}(D)\times\exp[-\bar\alpha BD^{-1}C\alpha]}$$

$\bar J = \bar\alpha B \quad j = C\alpha$

$$= \int\mathcal{D}[\bar\alpha,\alpha]\,e^{\bar\alpha(A-BD^{-1}C)\alpha}\ \det D$$

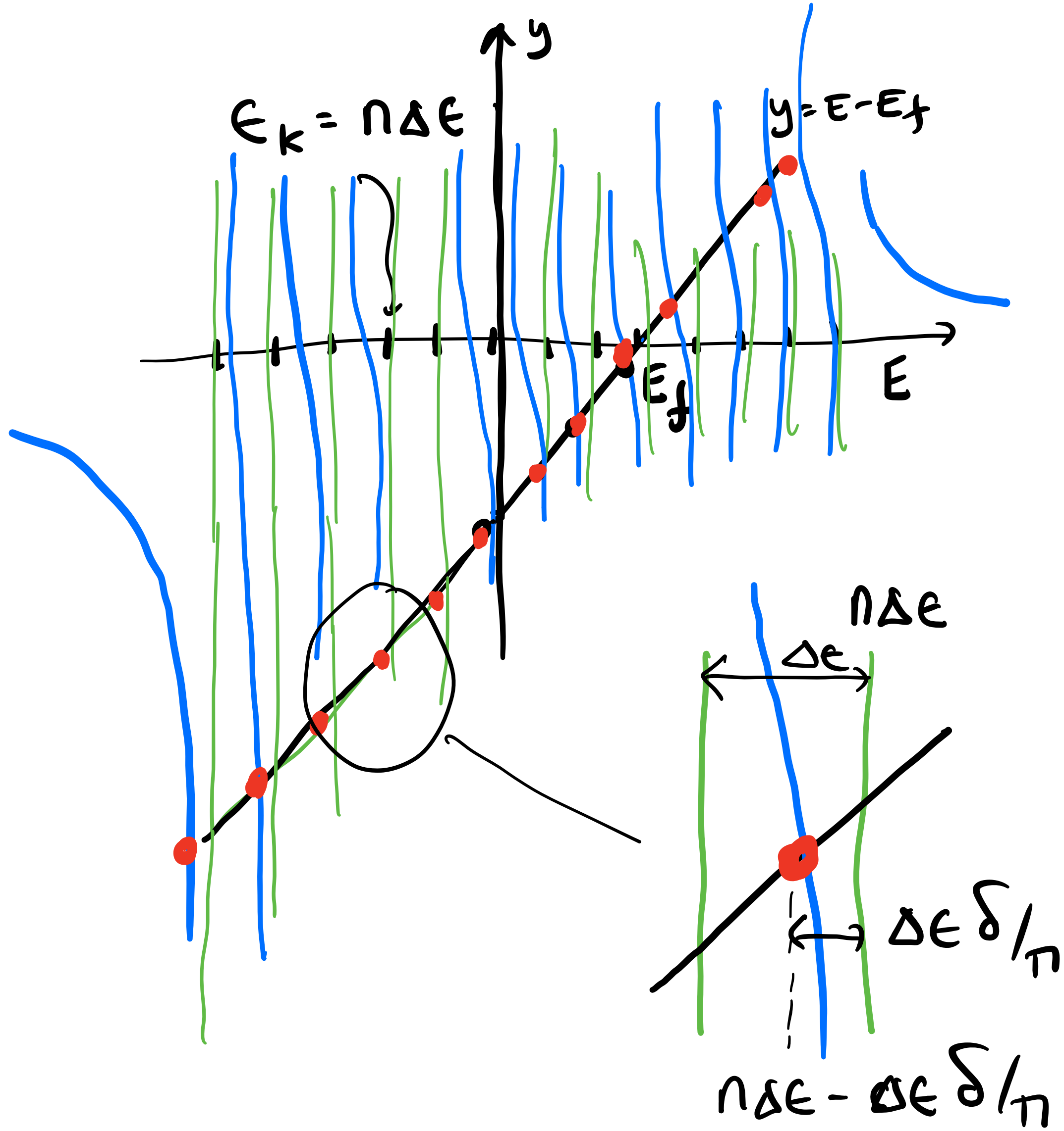
Relationship of phase shift with energy levels

$$G(\omega) = \frac{1}{\omega - E_f - \sum \frac{V^2}{\omega - \epsilon_K}}$$

Poles at $G^{-1}(E) = 0$

$$E - E_f = \sum \frac{V^2}{E - E_K}$$

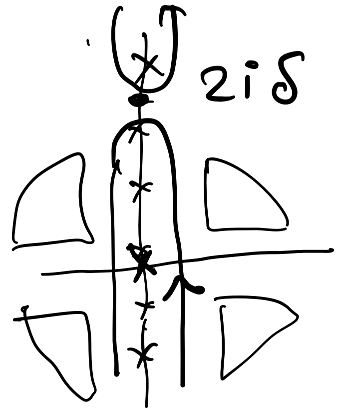
$$\bar{E} = n\Delta\epsilon - \frac{\delta}{\pi}(E)\,\Delta\epsilon$$



$$E - E_f = \frac{V^2}{\Delta\epsilon}\sum_n \frac{1}{(m-n) - \delta/\pi}$$

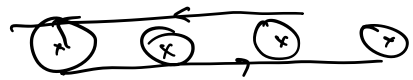
$$= \frac{V^2}{\Delta\epsilon}\sum_{n=-\infty}^{\infty}\frac{1}{(n-\delta/\pi)} = -\frac{V^2}{\Delta\epsilon}(\cot\delta)$$

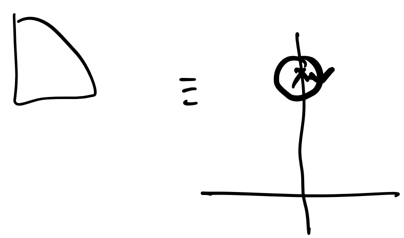
$$\sum \frac{1}{(n-\delta/\pi)} = \mathrm{Re}\oint \frac{dz}{2\pi i}\, n(z)\, \frac{e^{z0^+}}{\frac{z}{2\pi i} - \delta/\pi}$$



$$= \mathrm{Re}\oint dz\, n(z)\, \frac{e^{z0^+}}{z - 2i\delta}$$

$$\frac{1}{e^z - 1} - \frac{1}{2} \qquad z = 2\pi i n$$





$$\frac{1}{e^{z/2\pi i} - 1} \qquad z = n$$

$$\frac{e^{z/2} + e^{-z/2}}{e^{z/2} - e^{-z/2}}$$

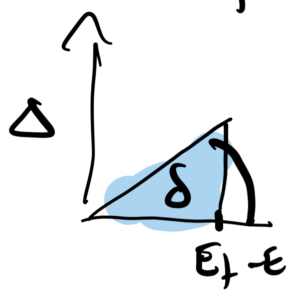
$$= -2\pi i\left(n(2i\delta) - \tfrac{1}{2}\right)$$

$$= -2\pi i\,\frac{1}{e^{2i\delta} - 1} = -\pi e^{-i\delta}\frac{2i}{e^{i\delta} - e^{-i\delta}} \Rightarrow -\pi\cot\delta$$

$$E - E_f = -\pi V^2 \rho \cot \delta$$

$$\delta = \cot^{-1}\left(\frac{E_f - E}{\Delta}\right) = \tan^{-1}\left(\frac{\Delta}{E_f - E}\right)$$

$$E_f + i\Delta - E = |E_f + i\Delta - E| \, e^{i\delta}$$



$$\cot \delta_f(\omega) = \frac{E_f - \omega}{\Delta} \Rightarrow \tan\delta = \left(\frac{\Delta}{E_f - \omega}\right) \qquad \delta = \mathrm{Im}\ln\left(E_f + i\Delta\right)$$

$$E_f + i\Delta = \sqrt{E_f^2 + \Delta^2}\, e^{i\delta}$$

$$\langle n_f \rangle = 2\int_{-\infty}^{0} \frac{d\omega}{\pi} A_f(\omega) = 2\,\mathrm{Im}\int_{-D}^{0} \frac{d\omega}{\pi} \frac{1}{\omega - E_f - i\Delta} = \frac{2}{\pi}\mathrm{Im}\ln\left[\frac{+E_f + i\Delta}{+D}\right]$$

$$= \frac{2}{\pi}\delta_f(\omega=0) = \sum \frac{\delta_\sigma}{\pi} \qquad \text{"Friedel sum rule"}$$

$$\boxed{f_\sigma(\tau) = \frac{1}{\sqrt{\beta}}\sum f_{\sigma n} e^{-i\omega_n\tau}}$$

$$S_F^0 = \sum_{\sigma, n} \bar{f}^{\dagger}_{\sigma n}\left[-G_f^{-1}(i\omega_n)\right] f_{\sigma n} = \sum \bar{f}_{\sigma n}\Big[E_f \underbrace{-i\Delta\,\mathrm{sgn}(\omega_n)}_{\Sigma(i\omega_n)} - i\omega_n\Big] f_{\sigma n}$$

$$= \int d\tau\, d\tau'\, \bar{f}_\sigma(\tau)\left[-\mathcal{G}^{-1}(\tau,\tau')\right] f_\sigma(\tau')$$

$$S_F = S_F^0 + U\int d\tau\, n_{f\uparrow} n_{f\downarrow}$$

$$\boxed{U n_\uparrow n_\downarrow \to U n_\uparrow \langle n_\downarrow\rangle + U n_\downarrow \langle n_\uparrow\rangle - U\langle n_\uparrow\rangle\langle n_\downarrow\rangle}$$

$$\to S_F^0 + \int\left[\phi_\uparrow n_\uparrow + \phi_\downarrow n_\downarrow - \frac{\phi_\uparrow\phi_\downarrow}{U}\right] d\tau.$$

$$-G_{f\sigma}^{-1} = \left[E_f + \phi_\sigma - i\Delta\,\mathrm{sgn}(\omega_n) - i\omega_n\right] \qquad \phi_\sigma = U\langle n_{f-\sigma}\rangle$$

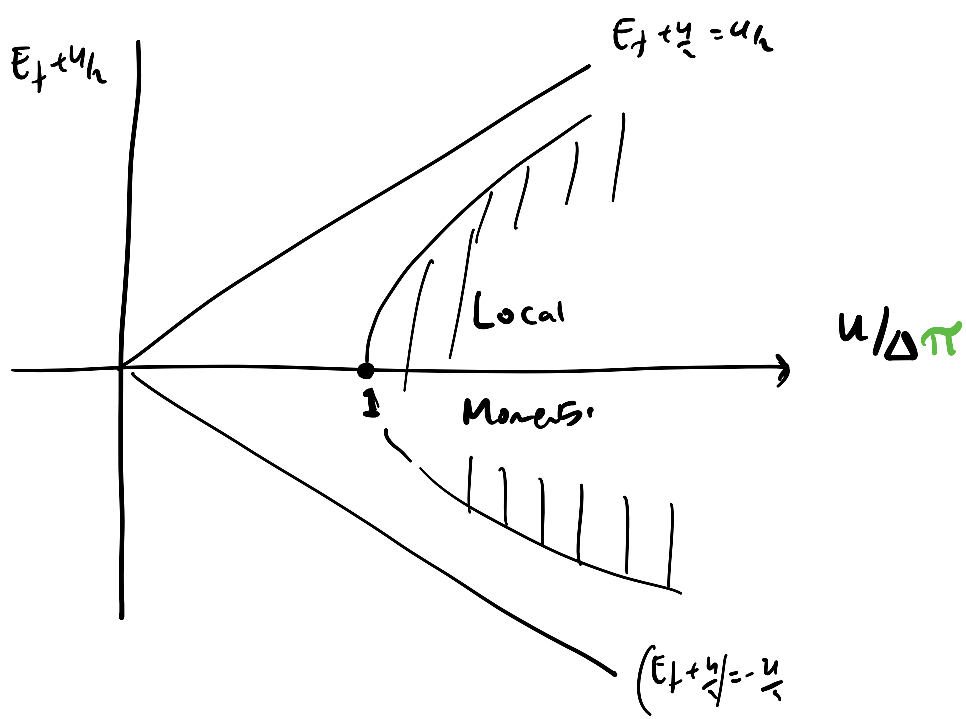
$(\sin\delta)$

$$\frac{\delta S_F}{\delta \phi_\sigma} = \langle n_\sigma \rangle - \frac{\phi_{-\sigma}}{U}.$$

$$\langle n_\sigma \rangle = \delta_\sigma / \pi = \frac{1}{\pi} \tan^{-1}\left[\frac{\Delta}{E_f + \phi_\sigma}\right] = \frac{\phi_{-\sigma}}{U}.$$

$$\phi_\sigma = \lambda + \sigma H \qquad n_{f\uparrow} = n_{f\downarrow} = n_f/2 = \frac{1}{\pi}\tan^{-1}\frac{\Delta}{E_f + \frac{U n_f}{2}}$$

$H = UM/2$
$\lambda = U n_f/2.$



$$\frac{1}{\pi}\tan^{-1}\frac{\Delta}{E_f + \frac{U n_f}{2} \mp H} = \frac{n_f}{2} \pm \frac{H}{U}$$

$$\frac{\pm H}{\Delta\pi\left(1 + \left(\frac{E_f + U n_f/2}{\Delta}\right)^2\right)} = \frac{\pm H}{U}$$

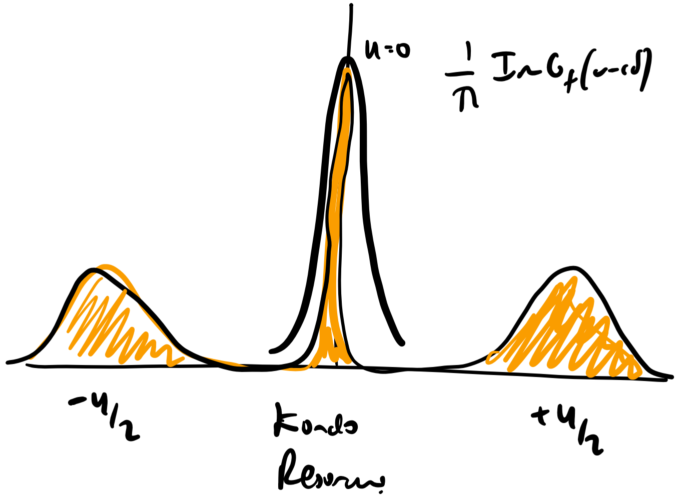
$\cot^2$

$$= \pm\frac{1}{\pi\Delta}\sin^2\delta_f$$

$$\frac{U}{\pi\Delta}\sin^2\delta_f = 1$$

$$\frac{U}{\pi\Delta} = \frac{1}{\sin^2\delta_f};$$

$$\left(E_f + \frac{U\delta}{\pi}\right)/\Delta = \cot\delta$$



$$\frac{1}{\pi}\mathrm{Im}\,G_f(\omega=0) = \left(\frac{\sin^2\delta}{\pi\Delta}\right)$$ Adiabatic Inv

$$\Delta\cot\delta = E_f + \frac{U\delta}{\pi}$$

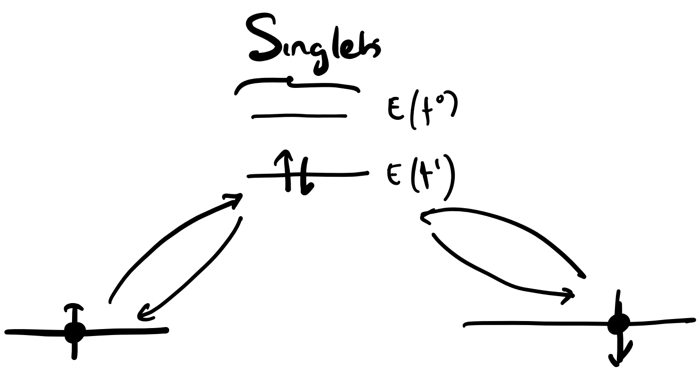
$$\Delta\cot\delta - \frac{U\delta}{\pi} = E_f$$

$$\frac{\pi\Delta}{2\sin^2\delta} + \Delta\cot\delta - \frac{\delta}{\pi}\frac{\Delta\pi}{\sin^2\delta} = E_f + U/2$$

$$E_f + U/2 = \Delta\cot\delta + \frac{\Delta\pi}{\sin^2\delta}\left(\frac{1}{2} - \frac{\delta}{\pi}\right)$$

$$\mathrm{Im}\,G_f(\omega=0) = \mathrm{Im}\,\frac{1}{-E_f - i\Delta}$$

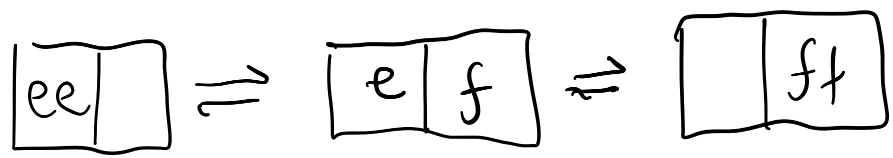
$$E_f + i\Delta = e^{i\delta}\sqrt{\Delta^2+E_f^2} \qquad = \left(\frac{\sin^2\delta}{\Delta}\right)$$



Virtual Charge fluctuations

$$e_\uparrow + f^1_\downarrow \rightleftharpoons f^2 \rightleftharpoons e_\downarrow + f^1_\uparrow \qquad \Delta E_I = U + E_f$$

$$\underbrace{e_\uparrow + f^1_\downarrow}_{\text{singlet}} \rightleftharpoons \underbrace{e_\uparrow + e_\downarrow}_{\text{singlet}} \rightleftharpoons e_\downarrow + f^1_\uparrow \qquad \Delta E_{II} = -E_f$$



$c^\dagger_\uparrow c^\dagger_\downarrow$ $\qquad (c^\dagger_\uparrow f^\dagger_\downarrow + c^\dagger_\downarrow f^\dagger_\uparrow)/\sqrt{2}$ $\qquad f^\dagger_\uparrow f^\dagger_\downarrow$

$$\Delta E = -2J \sim -2V^2 \left[\frac{1}{\Delta E_I} + \frac{1}{\Delta E_{II}} \right] = -2V^2 \left[\frac{1}{-E_f} + \frac{1}{E_f + U} \right]$$

P_{singlet}

$$(S_1 + S_2)^2 = 0$$

$$\frac{3}{2} + 2 S_1 \cdot S_2 = 0 \quad \begin{cases} 2 \\ 0 \end{cases}$$

$$= 1/4 \text{ trpl}$$

$$S_1 \cdot S_2 = -3/4 \text{ s.l}$$

$$\left[-(S_1 \cdot S_2) + 1/4 \right] P$$

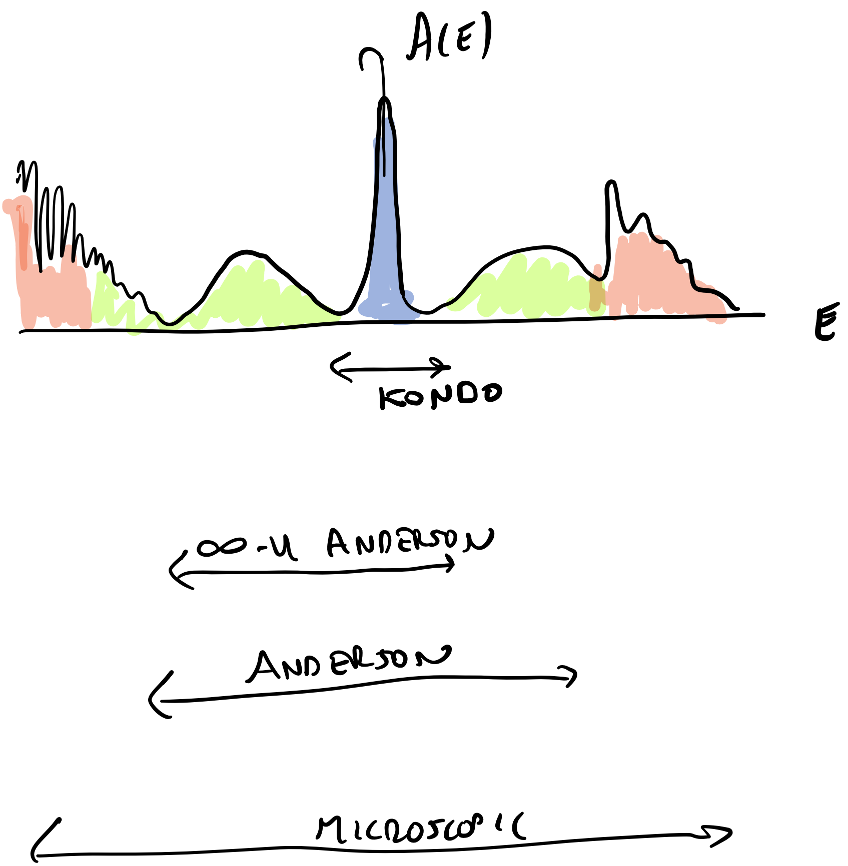
$$-\left(1/4 - \frac{\vec{S} \cdot \vec{\sigma}}{2} \right] (-2J) \equiv J \vec{S} \cdot \vec{\sigma}$$

$$\boxed{H = \sum \epsilon_k c^{+}_{k\sigma} c_{k\sigma} + J \vec{S} \cdot \vec{\sigma}(0)}$$

L3 THE KONDO EFFECT

- Renormalization.

$$H(D) = \left[\begin{array}{c|c} H_L & V^\dagger \\ \hline V & H_H \end{array}\right]$$



$$H(D) \to \tilde{H}(D) = U H U^\dagger$$

$$= \begin{bmatrix} \tilde{H}_L & 0 \\ 0 & \tilde{H}_H \end{bmatrix}$$

$$H(D') = b\tilde{H}_L \qquad \left(\frac{D'}{D} = \frac{1}{b}\right)$$

$$\frac{\partial g_i}{\partial \ln D} = \beta(\{g_i\})$$