$|b\rangle = e^{\hat{b}^\dagger b}|0\rangle$

BOSON

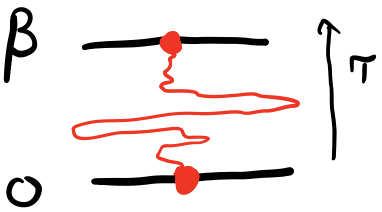
COHERENT STATE

BOSONS

FERMIONS

$|c\rangle = e^{\hat{c}^\dagger c}|0\rangle$

FERMION.

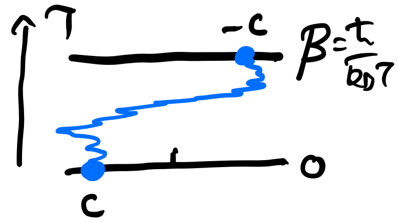


$b(\beta) = b(0)$

$$Z = \int \mathcal{D}[\bar{b}, b]\, e^{-S[\bar{b}, b]}$$

PATH INTEGRAL

$$S = \int_0^{\hbar/k_B T} \left(\bar{b}\,\partial_\tau b + H[\bar{b}, b]\right) d\tau$$



$c(\beta) = -c(0)$

$$\int \mathcal{D}[\bar{c}, c]\, \exp\left[-(\bar{c} M c - \bar{j} c - \bar{c} j)\right]$$
$$= [\det M]^{-\zeta} \exp\left[\bar{j} M^{-1} j\right]$$

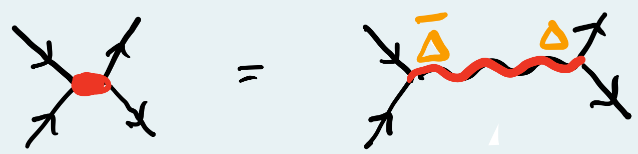
GAUSSIAN INTEGRAL

Free Particles

Feynman Diagrams

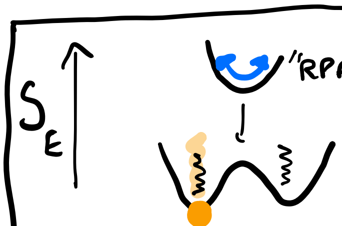
HUBBARD STRATONICH

$$-g A^\dagger A \longrightarrow A^\dagger \Delta + \bar{\Delta} A + \frac{\bar{\Delta}\Delta}{g}$$

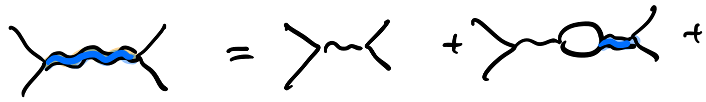


$$e^{-S_{EFF}[\Delta]} = \int \mathcal{D}[\bar{c}, c]\, e^{-S[\bar{c}, c, \Delta]}$$

EFFECTIVE ACTION



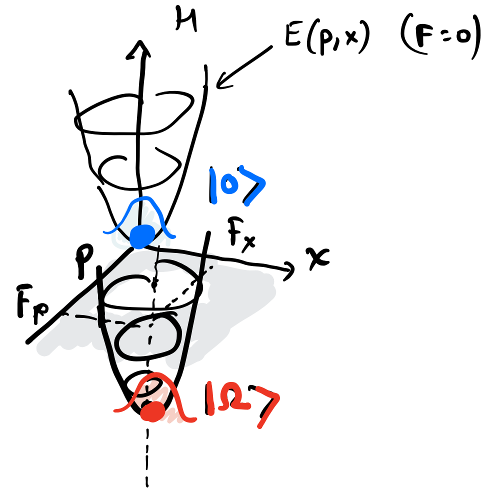
BROKEN SYMMETRY
GINZBURG LANDAU
ANDERSON HIGGS



RPA DESCRIPTIONS

Coherent States

$$e^{\hat{b}^\dagger b}|0\rangle = |b\rangle$$



e.g. $H = \frac{H'}{\omega_0} = \frac{p^2 + \hat{x}^2}{2} - F_x\hat{x} - F_p\hat{p}$.

$$x = (b+b^\dagger)/\sqrt{2},$$
$$p = \frac{b-b^\dagger}{\sqrt{2}i}.$$

The "source terms" F_x & F_p force the ground state into a coherent state located at $x+ip = F_x + iF_p$

$$\frac{H'}{\omega} \equiv H = \left(b^\dagger b + \tfrac{1}{2}\right) - \frac{F_x}{\sqrt{2}}(b+b^\dagger) - F_p\frac{(b-b^\dagger)}{\sqrt{2}i}$$

$$= b^\dagger b + \tfrac{1}{2} - (\bar{F}b + \bar{b}F)$$

$$= \Big[\underbrace{(b^\dagger - \bar{F})}_{a^\dagger}\underbrace{(b-F)}_{a} + \tfrac{1}{2} - \bar{F}F\Big]$$

$$\bar{F} = \frac{(F_x - iF_p)}{\sqrt{2}}$$
$$F = \frac{(F_x + iF_p)}{\sqrt{2}}$$

$$E_n = \left(\hat{n} + \tfrac{1}{2}\right) - \bar{F}F$$

$$\overbrace{(\hat{b} - F)}^{\hat{a}}|\Omega\rangle = 0 \Rightarrow \hat{b}|\Omega\rangle = F|\Omega\rangle \Rightarrow \boxed{|\Omega\rangle = e^{b^\dagger\frac{(F_x+iF_p)}{\sqrt{2}}}|0\rangle}$$

$$\frac{\langle\bar{\Omega}|b|\Omega\rangle}{\langle\bar{\Omega}|\Omega\rangle} = \left\langle\frac{x+ip}{\sqrt{2}}\right\rangle = \frac{F_x + iF_p}{\sqrt{2}}.$$

COMPLETENESS

$$|b\rangle = e^{\hat{b}^\dagger b}|0\rangle \qquad Tr[A] = \int d\bar{b}$$

$$\langle n|b\rangle = \frac{b^n}{\sqrt{n!}}$$

$$\hat{1} = \sum |n\rangle\langle n| = \int \frac{d\bar{b}\,db}{2\pi i} e^{-\bar{b}b} |b\rangle\langle\bar{b}|.$$

$$\delta_{nm} = \int \frac{d\bar{b}\,db}{2\pi i} e^{-\bar{b}b} \frac{\bar{b}^m b^n}{\sqrt{m!n!}} = \int 2r\,dr\; e^{-r^2} \frac{r^m r^n}{\sqrt{m!n!}} \underbrace{\int \frac{d\theta}{2\pi} e^{-i(m-n)\theta}}_{\delta_{mn}} = \int 2r\,dr\, e^{-r^2} \frac{r^{2m}}{m!} \delta_{nm}$$

$$b = re^{i\theta} \qquad \bar{b} = re^{-i\theta}$$

$$\delta\bar{b}\,db = \frac{\delta[\bar{b},b]}{\delta[r,\theta]} dr\,d\theta = \left\|\begin{matrix} \frac{\delta\bar{b}}{\delta r} & \frac{\delta\bar{b}}{\delta\theta} \\ \frac{\delta b}{\delta r} & \frac{\delta b}{\delta\theta}\end{matrix}\right\| dr\,d\theta = \begin{vmatrix} e^{-i\theta} & -ire^{-i\theta} \\ e^{i\theta} & ire^{i\theta}\end{vmatrix} dr\,d\theta = 2ir\,dr\,d\theta$$

$$= \int dx\, \frac{e^{-x} x^m}{m!} \delta_{nm} = \delta_{nm}.$$

$$\langle n|m\rangle = \langle n| \underbrace{\int \frac{d\bar{b}\,db}{2\pi i} e^{-\bar{b}b} |b\rangle\langle\bar{b}|}_{1} |m\rangle$$

- $$\boxed{1 = \int \frac{d\bar{b}\,db}{2\pi i} e^{-\bar{b}b} |b\rangle\langle\bar{b}| = \sum_{\bar{b}b} |b\rangle\langle\bar{b}|}$$

<u>Completeness</u>

e.g. $f(\alpha) = \langle f|\alpha\rangle$

$$f(\alpha) = \int \langle f|b\rangle\langle\bar{b}|\alpha\rangle \frac{d\bar{b}\,db}{2\pi i} e^{-\bar{b}b} = \int \frac{d\bar{b}\,db}{2\pi i} f(b)\, e^{\bar{b}(\alpha-b)}$$

$$= \int db\; \delta(\alpha-b)\, f(b)$$

$$\delta(\alpha-b) = \int \frac{d\bar{b}}{2\pi i} e^{\bar{b}(\alpha-b)}$$

- $$\boxed{Tr[\hat{A}] = Tr\Big[\sum_{\bar{b},b} |b\rangle\langle\bar{b}|\hat{A}\Big] = \int \frac{d\bar{b}\,db}{2\pi i} e^{-\bar{b}b} \langle\bar{b}|\hat{A}|b\rangle}$$

<u>Trace</u>.

$$\mathrm{Tr}\, e^{-\beta\hat{H}} = \int \frac{d\bar b_N db_0}{2\pi i} e^{-\bar b_N b_0} \langle \bar b_N | e^{-\beta H} | b_0\rangle$$

$$= \int \frac{d\bar b_N db_0}{2\pi i} \langle \bar b_N | \left(e^{-\Delta\tau H}\right)^N | b_0\rangle e^{-\bar b_N b_0}$$

$$\boxed{\hat 1_j = \int \frac{d\bar b_j db_j}{2\pi i} |b_j\rangle e^{-\bar b_j b_j}\langle \bar b_j|}$$

$$\boxed{\langle \bar b_j | e^{-\Delta\tau H}|b_{j-1}\rangle}$$

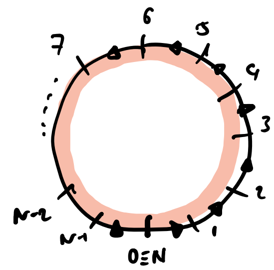
$$Z = \int \frac{d\bar b_N db_0}{2\pi i} \langle \bar b_N| e^{-\Delta\tau H} \times \dots\dots 1_j \times e^{-\Delta\tau H} \times 1_{j-1} \times \dots\ 1_1 e^{-\Delta\tau H}|b_0\rangle e^{-\bar b_N b_0}$$

$$= \int \mathcal{D}_N[\bar b, b] \prod_{j=1}^{N} e^{-\bar b_j b_j} \langle \bar b_j | e^{-\Delta\tau H}|b_{j-1}\rangle$$

$$\mathcal{D}_N[\bar b, b] = \prod_{j=1}^{N} \frac{d\bar b_j db_j}{2\pi i}$$

$b_N \equiv b_0 \qquad \bar b_N \equiv \bar b_0$

PERIODIC BCS



IMAGINARY TIME IS PERIODIC.

$$= \int \mathcal{D}_N[\bar b, b]\, e^{-\bar b_j b_j}\, e^{-\Delta\tau \epsilon \bar b_j b_{j-1}}\, e^{\bar b_j b_{j-1}}$$

$$= \int \mathcal{D}_N[\bar b, b]\, \exp\left[\sum_j \Delta\tau \left[\bar b_j \left(\frac{b_j - b_{j-1}}{\Delta\tau}\right) + H[\bar b_j, b_{j-1}]\right]\right]$$

$$\mathcal{D}_N[\bar b, b] \longrightarrow \mathcal{D}[\bar b, b]$$

$$b_j \to b(\tau) \qquad \bar b_j \frac{(b_j - b_{j-1})}{\Delta\tau} \to \bar b\, \partial_\tau b$$

$$H[\bar b_j, b_{j-1}] = H[\bar b, b]$$

$$\boxed{\begin{aligned} Z = \mathrm{Tr}\, e^{-\beta H} &= \int \mathcal{D}[\bar b, b]\, e^{-S} \\ S &= \int_0^\beta d\tau \left(\bar b\, \partial_\tau b + H[\bar b, b]\right) \end{aligned}}$$

$$\boxed{\begin{aligned} b(\beta) &= b(0) \\ \bar b(\beta) &= \bar b(0) \end{aligned}}$$

$H = \epsilon b^\dagger b$

$$Z = \int \mathcal{D}[\bar b, b]\, \exp\left[-\int_0^\beta d\tau \int \bar b(\partial_\tau + \epsilon) b\right]$$

NON LINEAR OSCILLATOR

$$h = \epsilon b^\dagger b + g :(b + b^\dagger)^4:$$

$$\hookrightarrow b^4 + 4 b^\dagger b^3 + 6 (b^\dagger)^2 b^2 + 4 (b^\dagger)^3 b + b^{\dagger 4}.$$

$$\boxed{\begin{aligned} Z &= \int \mathcal{D}[\bar{b}, b]\, e^{-S} \\ S &= \int_0^\beta d\tau \left[\bar{b} (\partial_\tau + \epsilon) b + g (b + \bar{b})^4 \right] \end{aligned}}$$