INTRODUCTION TO MANY BODY PHYSICS: 620. Fall 2011

Questions 2. (Due Fri, Sept 30)

1. In this question $c_i{}^\dagger$ and c_i are fermion creation and annihilation operators and the states are fermion states. Use the convention

   $|11111000\ldots\rangle = c_5{}^\dagger c_4{}^\dagger c_3{}^\dagger c_2{}^\dagger c_1{}^\dagger|\text{vacuum}\rangle$.

   (i) Evaluate $c_3{}^\dagger c_6 c_4 c_6{}^\dagger c_3|111110000\ldots\rangle$.

   (ii) Write $|1101100100\ldots\rangle$ in terms of excitations about the "filled Fermi sea" $|1111100000\ldots\rangle$. Interpret your answer in terms of electron and hole excitations.

   (iii) Find $\langle\psi|\hat{N}|\psi\rangle$ where $|\psi\rangle = A|100\rangle + B|111000\rangle$, $\hat{N} = \sum_i c_i{}^\dagger c_i$.

2. (a) Consider two fermions, a_1 and a_2. Show that the Boguilubov transformation

$$\begin{aligned} c_1 &= ua_1 + va^\dagger{}_2 \\ c^\dagger{}_2 &= -va_1 + ua^\dagger{}_2 \end{aligned} \tag{1}$$

   where u and v are real, preserves the canonical anti-commutation relations if $u^2 + v^2 = 1$.

   (b) Use this result to show that the Hamiltonian

$$H = \epsilon(a^\dagger{}_1 a_1 - a_2 a_2{}^\dagger) + \Delta(a^\dagger{}_1 a^\dagger{}_2 + \text{H.c.}) \tag{2}$$

   can be diagonalized in the form

$$H = \sqrt{\epsilon^2 + \Delta^2}(c^\dagger{}_1 c_1 + c^\dagger{}_2 c_2 - 1) \tag{3}$$

   (c) What is the ground-state energy of this Hamiltonian?

3. (i)Use the Jordan Wigner transformation to show that the one dimensional anisotropic XY model

$$H = -\sum_j [J_1 S_x(i) S_x(i+1) + J_2 S_y(i) S_y(i+1)] \tag{4}$$

   can be written as

$$H = -\sum_j [t(d^\dagger{}_{i+1} d_i + \text{H.c}) + \Delta(d^\dagger{}_{i+1} d^\dagger{}_i + \text{H.c})] \tag{5}$$

   where $t = \frac{1}{4}(J_1 + J_2)$ and $\Delta = \frac{1}{4}(J_2 - J_1)$.

   (ii)Calculate the excitation spectrum for this model and sketch your results. Comment specifically on the two cases $J_1 = J_2$ and $J_2 = 0$.