Physics 618 Twenty-sixth Lecture April 25, 2017

Most of this course was concerned with the finite-dimensional unitary representations of finite or compact semisimple symmetry groups, as those representations are suitable for describing the energy eigenstates of the quantum system with finitely-many degrees of freedom. But then we turned to other considerations, in particular to field theories and the possibility of local symmetries. We found very different physical effects of the symmetry groups, including gauge bosons, Goldstone bosons, and Higgs-induced massive vector bosons in broken symmetries. We finished our discussion of Higgs with the standard model, adding in fermions to the SU(2)×U(1) bosons and discussing how, in a model that requires all the fundamental fields to enter the Lagrangian as massless particles, we can end up with all but the photon, and perhaps some neutrinos, developing mass.

We then changed topics, beginning a path which will lead to supersymmetry, but which begins with just Einstein's special-relativistic version of Galilean inertial coordinate equivalence. We have translational invariance, rotational invariance, and the equivalence of coordinate systems in uniform motion with respect to each other, just as Galileo said, except for incorporating time in a more integral way, preserving a finite speed of light.

Using the $x^0 = ct$ as the fourth coordinate of space-time, we saw that the symmetry involved is that the laws of physics are the same whether expressed in terms of x^μ or in terms of $x'^\mu = \sum_\nu a^\mu{}_\nu x^\nu + b^\mu$, as long as $a^\mu{}_\nu$ is a Lorentz transformation satisfying $\eta_{\mu\rho} a^\mu{}_\nu a^\rho{}_\sigma = \eta_{\nu\sigma}$, with $\eta = \operatorname{diag}(-1,1,1,1)$, which ensures that the spacetime "distances" $(ds)^2 = (d\vec{x})^2 - c^2(dt)^2$ are invariant. We found that the Lie algebra generators of this Poincaré group are the momenta P^μ and the Lorentz transformations $L^\mu{}_\nu$ satisfying

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[L_{\mu\nu}, P_\rho] &= -i\eta_{\mu\rho} P_\nu + i\eta_{\nu\rho} P_\mu \\
[L_{\mu\nu}, L_{\rho\sigma}] &= -i\eta_{\nu\rho} L_{\mu\sigma} + i\eta_{\nu\sigma} L_{\mu\rho} \\
&\quad + i\eta_{\mu\rho} L_{\nu\sigma} - i\eta_{\mu\sigma} L_{\nu\rho},
\end{aligned}
$$

with $L_{\mu\nu} := \eta_{\mu\rho} L^\rho{}_\nu$. This gives us an algebra which gives us the invariant Casimir operators

$$
\begin{aligned}
P^2 &:= \eta^{\mu\nu} P_\mu P_\nu \\
W^2 &:= \eta_{\mu\nu} W^\mu W^\nu \quad \text{with} \\
&\quad W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} P_\nu L_{\rho\sigma}.
\end{aligned}
$$

We saw that for a state with timelike P^μ, that is, with energy greater than the momentum, P^2 is the square of the invariant mass and in the rest frame, with $P^\mu = (m,0,0,0)$, $W^j = mL_j$, with L_j the angular momentum in the (spatial) j direction.

Today

Of course irreducible representations of a symmetry must have definite values for the Casimir operators, so the irreducible representation of the

Poincaré group will have definite mass m and definite half-integral spin, but as we will see for the isochronous proper group we will have left and right handed spins. We will review the consequences, in particular for the spin 1/2 representations.

Then we will turn to the question of whether it is possible to combine the Poincaré symmetry with internal symmetries, such as isopin, flavor SU(3), or color, in a non-trivial way, which is to say, in a bigger group that includes the direct product as a proper subgroup. First we will find it is impossible, and then we will do it anyway, with the invention of supersymmetry. We will discuss the possibility of combining all four fundamental interactions into a supergravity theory, and also develop the notion of a superfield.

Reminder:

> The Final exam is on May 4 at noon (to 3:00 PM) in SEC room 206. Unless you convince me otherwise now, it will be closed book, but you may bring in 3 sheets of handwritten notes.