We are now about to consider yet another way in which symmetries are incorporated into our physics, probably the oldest, the symmetry of space and time. Without translational symmetry there would be no point doing physics, because results from my lab would be irrelevant to what happens in your lab. Same for translational invariance in time, or today’s experiments would tell us nothing about what will happen in the future. Rotational invariance about the vertical axes is built into everyone’s intuition, but many Greeks did think down was different. We now know that it is better to start our physics with empty space. In addition to four-dimensional translational invariance and three-dimensional rotational invariance, Galileo taught us all inertial frames are equivalent, so mixing spatial coordinates and time. Of course Einstein made some minor modifications with the special theory of relativity. This leads us to the Poincaré group.

The Poincaré group is a 10 dimensional Lie group, but it is neither simple nor semisimple nor compact, so its representations will be a bit different from what we discussed in the bulk of this course. The representations will be quantum fields. Restricting attention to the Lorentz subgroup, that is, ignoring the translations, we will find finite dimensional representations, which then coupled with wave functions provide the fields. In particular, we will talk a bit about spinors.

We will detail the Lie algebra of the Poincaré group, which consists of 4 momenta and 6 Lorentz transformations, three of which are rotations, and three are Lorentz boosts. The momenta form an Abelian subalgebra, the 1-dimensional irreducible representations of which are labelled by the 4-momentum $P^\mu$. We will look for Casimir operators of the full Poincaré group. These are the operators which commute with all the generators of the Lie algebra and therefore take on a single value for any state in an irreducible representation. These will give us the mass and the spin of the state.

The states of a given momentum will be represented by functions of $x^\mu$, which is to say fields, but we will explore how the Lorentz transformations require our fields to be in multiplets described by a pair of spins. We have already assumed this in all our discussion of elementary particles. But we also saw there that other symmetries, involving internal variables such as isospin or color, play a crucial role in elementary particle physics. We will entertain the question of whether these two forms of symmetry, Poincaré and internal, can be combined into a more powerful symmetry. At first we will be stymied by the no-go theorem of Coleman and Mandula, which states that no sensible theory has a Lie group symmetry which combines the Poincaré group and an internal symmetry group in a way other than a direct product, which is to say, the symmetries are uncoupled. But then we will find a way to circumvent that theorem by generalizing our concept of a symmetry algebra, introducing supersymmetry.

**Announcements**

Normal classes this week.

Next week, classes on Tuesday as usual, and the last class on Wednesday, at noon, in ARC 212.

There will be no more homeworks.

Final exam is scheduled for Thursday, May 5th, at noon, unless we want to change it to another time. Do we?