We notice that this is not the sum of $P^2_\ell$'s but instead the sum of $iU_\ell P_\ell$ squared. So we will define electric field $E_\ell = iU_\ell P_\ell$ in terms of which the Lagrangian is $\propto \text{Tr} E$. 

We will then be a little more careful, noting that for SU(N) not all elements of $U$ are unconstrained. With greater care we should have taken $i\omega$ as real parameters, and define the quantum elements of $U$ in terms of the group elements coming from parallel transport along links at time $t=0$ unchanged but convert each time link to the identity, to find a gauge-equivalent configuration of links with all time links set to $\mathbb{I}$. This is known as the temporal gauge. Then the placquette in the $(0,j)$ plane has 

$$U_{P_{ij}} = U_{n\ell,0}^{a} U_{n\ell+aq, j}^{a} U_{n\ell+aq,0} U_{n,j}.$$ 

As the time lattice spacing goes to zero, we may Taylor expand $U_{n\ell +aq,j}^{a}$ to second order in the lattice spacing $a^i$, and similarly for $U_{n\ell +aq,j}^{a}$. When adding, $U_P + U'_P - 2$, the first order terms are $a^i \frac{\partial}{\partial \Pi_i} (U^{-1}U) = 0$, and the second order piece can be reexpressed as $\text{Tr} (U_j^{-1}U_j U_j^{-1}U_j)$. The space-space placquettes will give a potential term $V(U_{P_{jk}})$ without time derivatives. Thus 

$$L = \sum_{\ell} \frac{-a}{2g^2} \text{Tr} (U^{-1}_\ell \dot{U}_\ell U^{-1}_\ell \dot{U}_\ell) - V(\{U\}).$$ 

We will concentrate on the first term, trying to understand the kinetic terms that do involve time derivatives, and hence the canonical momenta. The canonical coordinates are matrices on each link, $U_{\ell ab}$, where $\ell$ specifies the link, both position and spacelike direction, and $a, b$ are indices of Lie algebra generators.

**Today**

We will begin today with a naïve definition of the canonical momentum 

$$(P_\ell)_{ab} := \frac{\delta L}{\delta (\dot{U}_\ell)_{bc}} = \frac{-a}{g^2} \left( U^{-1}_\ell \dot{U}_\ell U^{-1}_\ell \right)_{ab},$$

and then the Hamiltonian is 

$$H = \sum_{\ell, c, b} \dot{U}_{\ell bc} P_{\ell cb} - L = \sum_{\ell} \frac{-g^2}{2a} \text{Tr} (U_\ell P_\ell U_\ell P_\ell) + V(\{U\}).$$