Last time we discussed the extraction of arbitrary irreducible representations of SU($N$) from the tensor products of $k$ defining representations. The modules are a vector space with basis vectors $e_{a_1} \otimes e_{a_2} \otimes \ldots \otimes e_{a_k}$ with coefficient tensors $w^{a_1a_2\ldots a_k}$, with each $a_j = 1, \ldots, n$. Applying the identity decomposition $1 = \sum_{\eta} e_{\eta_{ii}}$ to $w$ decomposes it into pieces for each $e_{\eta_{ii}}$, each piece lying in an irreducible representation.

We then found a simple way to determine the dimension the each irreducible representation of SU($N$) corresponding to each Young graph in terms of products of numbers placed in the boxes.

Today

Today we will learn how to decompose the tensor product of two irreducible representations of SU($N$). This is especially important in the scattering of particles, because if the scattering matrix is SU(3) invariant, the scattering amplitudes can be decomposed using Wigner-Eckhart theorem into a smaller number of reduced matrix elements. But we will find a complication which did not arise in SU(2). The decompositions and corresponding Clebsch-Gordon (physics version) coefficients are also important in understanding the partial decay rates of hadronic resonances.

The tensor products also occur when considering that the quantum state depends on several variables for each of the components. For example, a baryon consisting of three quarks depends on the color, flavor, and spin of each of the three quarks, even if there is no orbital angular momentum. We shall see that this gives good agreement with observed light baryons only if color is taken into effect.

We are now about to make a big transition in how to apply group theory to physics. The discrete symmetry transformations we have considered, largely in classifying states of a system, is very useful, but not nearly as sexy as the idea of gauge symmetry groups, which we now understand as underlying all of fundamental physics. The basic idea is that there are fields, in the physicists sense of functions on space-time, which have symmetry properties independently at each space-time point, a local symmetry.

But before we get to that, I think you are not all familiar with the dynamics of fields, in particular the lagrangian mechanics of distributed degrees of freedom. I will first describe the dynamics of a massless string under tension with a large number of point masses on it, which you know how to apply ordinary lagrangian mechanics, and take a continuum limit of this to get the lagrangian approach to a massive string, the simplest (one-dimensional) field theory. Then we will expand to three spacial dimensions and see that
time comes in naturally as a fourth component. My lecture notes for this are added ad-hoc from my lecture notes for classical mechanics, Physics 507, which you can find in full at

http://www.physics.rutgers.edu/~shapiro/507/gettext.shtml

but I have posted an extraction from that which should be all you need for the next lecture. If we take the local degrees of freedom to be a set of scalars $\eta_i$, and the Lagrangian density to be $L = \frac{1}{2} \sum_i \left( \sum_\mu (\partial_\mu \eta_i)(\partial^\mu \eta_i) + m^2 \eta_i^2 \right)$ we get the equations of motion $(\partial_t^2 - \nabla^2 - m^2) \eta_i = 0$, the Klein-Gordon equation for a free particle of mass $m$ (with $\hbar = c = 1$).

Reminder:

Eighth Homework due Friday, April 1 at 4:00 PM
Next week: classes Tuesday and Wednesday, not Friday.

To discuss: Do we need an introduction to field theory?