Before the Break

We were looking to find all the finite-dimensional representations of $SU(N)$, and we observed that for $N > 3$ there are too many sets of indices to fit comfortably on a tensor, but instead we could generate all representations using only the defining representation if we allowed for fancy permutation symmetries. So we began our discussion of representations of the permutation group $S_k$.

We defined the group algebra of any finite group as a formal sum of the group elements multiplied by complex coefficients. The elements of the group algebra of $S_k$ don’t have an intuitive meaning by themselves, but their action on tensors with $k$ indices is well defined as these form a vector space. In particular, given a function $f$ on $k$ variables of the same type, \( \frac{1}{k!} \sum_P P f \) is the totally symmetric part of $f$, and \( \frac{1}{k!} \sum_P (-1)^P P f \) is the totally antisymmetric part of $f$, but extracting the other parts requires other elements of the $S_k$ group algebra, which are related to irreducible representations of $S_k$.

The conjugacy classes of $S_k$ are given by partitions of $k$. We defined Young graphs and Young tableaux for these partitions $\eta$, and found group algebra elements $e_{\eta}^{ij}$ which form projection operators onto ideals. We found that the number of the representation $\eta$ is the number of standard tableaux and this is given by a magic algorithm involving the product of hooks in the boxes of the Young graph.

Today

Today we will apply this arsenal of algebraic algorithms to explore the representations of $SU(n)$. Starting with the tensor product of $k$ copies of the defining $\mathbf{N}$ representation, with coefficients a tensor with $k$ indices, we have a reducible representation with $N^k$ degrees of freedom, but applying the projection operator $e_{\eta}^{ij}$ to that projects out many, leaving us with an irreducible representations for each $\eta$ and $j$, whose dimension we will want to count.

By the way, an exhaustive treatment of this material is in an old self-published book by Schensted, still available, but for about 10 times as much as I paid. See the Book Refs web page.

Announcements:
I feel a bit guilty about not guiding you through the $q - p$ box drawing methods of Georgi in constructing the full adjoint representation from the
Cartan matrix. I have added a note to the Supplementary Notes web page which gives what I think is an improved diagrammatic prescription, in *On the q-p diagrams of Georgi*. You will not be responsible for this material, but I include it for completeness.

Homework 7 due on Thursday.

Classes as usual this week.

Next week, class on Friday is moved to Wednesday, March 30, in ARC 212.