Last time we saw how isospin SU(2) gave us cross sections of many different $\pi$-nucleon semi-elastic scattering in terms of only two functions, an example of the Wigner-Eckhart theorem decomposition into math versus physics questions. We also saw the beginning of the quark model and the suggestion that groups larger than SU(2) might be useful. So we switched to asking what we can say about all symmetry groups of this type, semisimple compact Lie groups and their representations. We started analyzing their representations by diagonalizing the maximum number of generators $H_i$, generating the Cartan subalgebra, and defining their eigenvalues $\vec{\mu}$ for all the other dimensions of the Lie algebra $L$ as the weights of the representation. We saw that the generators of these other dimensions $E_{\vec{\alpha}}$ acted as raising and lowering operators just as $L_{\pm}$ did for SU(2). Applying this to the adjoint representation, and calling the nonzero weights there to be roots $\vec{\alpha}$, we found the “master formula”

$$\frac{\vec{\mu} \cdot \vec{\alpha}}{\alpha^2} = \frac{q - p}{2}.$$

Applying this to the adjoint representation, we found that any two weights satisfied $\vec{\alpha} \cdot \vec{\beta}/\alpha^2 = n/2$ for $n \in \mathbb{Z}$, which restricted the angles between weights to just a few possibilities, and also the ratio of lengths.

Today

We will see that this constraint is extremely powerful, so that by the end of the week we will have found all the possible semisimple finite-dimensional compact Lie groups. But before doing so we will examine how this works in the next-simplest algebra, SU(3).

Reminders:

We meet again on Friday in 203. Next week classes are on the normal schedule.

There will be a midterm exam on Tuesday, March 8. I currently think this will cover the material from the beginning through Dynkin diagrams (Chapter 8), but we might modify that as we get closer to the exam.

Homework 5 is due tomorrow.

Homework 6 will be due Thursday, March 3. It has been posted.