

Copyright C 1984-2012 by Joel A. Shapiro

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, or otherwise, without the prior written permission of the author, except that students taking my course may print a copy for their own personal use, but may not sell it.

Note that these are lecture notes, not a publication, and the author acknowledges that much of this material is taken from the books of Joshi, Jones, and Georgi.

The author welcomes corrections, comments, and criticism.

Group Theory in Physics

Joel A. Shapiro

April 25, 2017

Contents

1	Groups	7
1.1	Introduction	7
1.2	Definition of a Group	11
1.3	Conjugacy	18
1.4	Cosets	19
1.5	Cyclic Group	20
1.6	Normal Subgroups	20
1.7	Direct Products	21
1.8	Kernel	23
1.9	Permutations	23
1.10	Groups of low order	25
2	Representations	27
2.1	Schur's First Lemma	31
2.2	Schur's Second Lemma	34
2.3	The Great Orthogonality Theorem	34
2.4	Characters	38
2.5	Examples	42
2.6	Point, Space, and Crystallographic Groups	46
2.7	Direct Products of Representations	46
3	Infinite Groups	49
3.1	Connectedness	50
3.2	Infinitesimal Generators	52
3.3	Adjoint Representation, Killing Form, <i>etc.</i>	61
3.4	Quantum Operators	64

4	SU(2)	67
4.1	Representations of SU(2)	67
4.2	Reduction of Direct Products	71
4.3	Representations of Finite Rotations	75
4.3.1	Isospin	77
5	Semisimple Compact Lie Groups	85
6	SU(3)	91
7	Simple Roots	95
8	Dynkin Diagrams	99
8.1	Classification of Simple Lie Algebras	100
9	Finding the Other Roots	105
10	Representations of Lie Groups	109
10.1	Fundamental Weights	109
10.1.1	SU(3) Multiplets	110
10.2	Tensor Methods	115
11	S_k and Tensor Representations	121
11.1	Irreducible Representations of S_k	122
11.2	Representations of SU(n)	127
12	Tensor Products of Irreducible Representations	133
13	Local Symmetry	137
13.1	Gauge Invariance	147
14	Hamiltonian Formulation of Local Symmetry	151
14.1	Relation of R_j to E_j	156
15	Phonons, Bloch Waves; Spontaneous Symmetry Breaking	163
15.1	Translations on a Lattice	163
15.2	Spontaneous Symmetry Breaking	166

16 More SSB, then Higgs 171

- 16.1 An example: SSB for SO(N) 171
- 16.2 The Higgs Mechanism 173
 - 16.2.1 The Abelian Higgs Model 174
 - 16.2.2 Broken Non-Abelian Gauge Theory 175
 - 16.2.3 A Side Comment on g 176
 - 16.2.4 SU(2)×U(1) Gauge Theory with Isodoublet Higgs . . . 177
- 16.3 Adding Leptons and Quarks 178
 - 16.3.1 Quark Weak Interactions 180
 - 16.3.2 Fermion Masses 182

17 Poincaré and Susy 185

- 17.1 Poincaré Invariance 185
- 17.2 Supersymmetry 190
 - 17.2.1 Superparticle Multiplets 193
- 17.3 Superfields 196

18 Conformal Invariance 203

- 18.1 Maxwell’s Equations 207