Physics 618 Homework #8

Due: Thursday, April 6, 2017 before 4:00 PM

1 [10 pts] Consider the representations $\ydiagram{3}$ and $\ydiagram{2,1}$ for $SU(3)$, $SU(4)$, and $SU(N)$.

a) Find the dimension of each of these representations for each group.

b) Decompose the product $\ydiagram{3} \otimes \ydiagram{2,1}$ into irreducible representations. For each irreducible representation, give its Young graph and its dimensionality for each group. Check that no dimensions (*i.e.* degrees of freedom) get lost.

c) For $SU(3)$, consider the scattering of mesons in the octet ($\pi^\pm, \pi^0, \eta$, $K^+, K^0, \bar{K}^0, \bar{K}^-$) with the spin 3/2 baryons ($\Delta^{++}, \Delta^\pm, \Delta^0, \Sigma^{*\pm}, \Sigma^{*0}, \Xi^{*0}, \Xi^{*-}, \Omega^-$), however unrealistic it may be. How many different scattering amplitudes would be needed to describe all the scattering processes of the 80 possible input states into the 80 possible output states, assuming the flavor $SU(3)$ is conserved.

2 [5 pts] Prove that, for any operator $A(x)$,

$$\frac{\partial}{\partial x} e^{A(x)} = \int_0^1 d\alpha e^{\alpha A(x)} \frac{\partial A(x)}{\partial x} e^{(1-\alpha)A(x)}.$$

[Hints: Expand all exponentials as power series. Use the fact that the beta function

$$B(z,w) = \int_0^1 d\alpha \alpha^{z-1}(1-\alpha)^{w-1} = \frac{\Gamma(z)\Gamma(w)}{\Gamma(z+w)}. \qquad]$$