Physics 618  Homework #8
Due: Friday, April 1, 2016 before 4:00 PM

[Note changed due date, as no class on Friday]

1. Construct $P_\tau$ and $Q_\tau$ for the two standard tableaux of the Young Graph

Construct $Q_i s_{ij} P_j$, where $s_{12} = s_{21} = (23)$, $s_{11} = s_{22} = I$. Show that this generates a four-dimensional subspace of the group algebra which is the same as that generated by $e_{ij}^\eta$, using my solution to Problem 1, Assignment 3.

2. For the representation of $S_4$, the characters are $\chi(I) = 2$, $\chi((123)) = -1$, $\chi((12)(34)) = 2$, $\chi((12)) = 0$, $\chi((1234)) = 0$. Find the dimension of the corresponding $SU(N)$ representation by counting independent tensors, and then verify this answer by using the magic counting formula (i.e. putting numbers in boxes).

3. Consider the representations and for $SU(3)$, $SU(4)$, and $SU(N)$.

a) Find the dimension of each of these representations for each group.
b) Decompose the product into irreducible representations. For each irreducible representation, give its Young graph and its dimensionality for each group. Check that no dimensions (i.e. degrees of freedom) get lost.
c) For $SU(3)$, consider the scattering of mesons in the octet ($\pi^\pm, \pi^0, \eta, K^+, K^0, \bar{K}^0, \bar{K}^-$) with the spin 3/2 baryons ($\Delta^{++}, \Delta^+, \Delta^0, \Sigma^{*+}, \Sigma^{*0}, \Xi^{*0}, \Xi^{*-}, \Omega^-$), however unrealistic it may be. How many different scattering amplitudes would be needed to describe all the scattering processes of the 80 possible input states into the 80 possible output states, assuming the flavor $SU(3)$ is conserved.