Physics 618 Homework #7

Due: Thursday, March 30, 2017 at 4:00 PM

1 [5 pts] [Note: this is Georgi's problem 9.A] If $|\mu\rangle$ is the state of the highest weight ($\mu = \mu^1 + \mu^2$) of the adjoint representation of SU(3), show that the states

$$\begin{aligned} & |A\rangle &=& E_{-\alpha^1} E_{-\alpha^2} |\mu\rangle \\ \text{and} \qquad & |B\rangle &=& E_{-\alpha^2} E_{-\alpha^1} |\mu\rangle \end{aligned}$$

are linearly independent.

Hint: Calculate the matrix elements $\langle A|A\rangle$, $\langle A|B\rangle$, $\langle B|A\rangle$ and $\langle B|B\rangle$. Show that $|A\rangle$ and $|B\rangle$ are linearly dependent if and only if

$$\langle A|A\rangle\langle B|B\rangle = \langle A|B\rangle\langle B|A\rangle.$$

2 [10 pts] Construct P_τ and Q_τ for the two standard tableaux of the Young Graph $\ydiagram{2,1}$. Construct $Q_i s_{ij} P_j$, where $s_{12} = s_{21} = (23)$, $s_{11} = s_{22} = 1\!\!1$. Show that this generates a four-dimensional subspace of the group algebra which is the same as that generated by e^η_{ij}, using my solution to Problem 1, Assignment 3.

3 [5 pts] For the $\ydiagram{2,2}$ representation of S_4, the characters are $\chi(1\!\!1) = 2$, $\chi((123)) = -1$, $\chi((12)(34)) = 2$, $\chi((12)) = 0$, $\chi((1234)) = 0$. Find the dimension of the corresponding $SU(N)$ representation by counting independent tensors, and then verify this answer by using the magic counting formula (*i.e.* putting numbers in boxes).