

## Some Comments on P&S treatment of BRST

### Below 16.49

That  $Q^2$  vanishes on  $B$  and on  $\bar{c}$  is trivial, the first killed by the first  $Q$  and the second converted by the first  $Q$  into a  $B$  and then killed by the second  $Q$ . For the fermion field

$$\begin{aligned}
 Q^2\psi &= Q(igc^at^a\psi) = igQ(c^a)t^a\psi - igc^at^aQ(\psi) \\
 &= ig\left\{-\frac{1}{2}gf^{abc}c^bc^ct^a\psi - igc^at^ac^bt^b\psi\right\} \\
 &= -\frac{1}{2}g^2\left\{if^{abc}c^bc^ct^a\psi - c^ac^b[t^a, t^b]\psi\right\} \\
 &= -\frac{1}{2}g^2\left\{if^{abc}c^bc^ct^a\psi - if^{abc}c^ac^bt^c\psi\right\} = 0.
 \end{aligned}$$

It is important to notice in the first line, that when the anticommuting  $Q$  passes through the grassman  $c^a$ , we pick up a minus sign. (I missed that in lecture).

### On the Subspaces

We saw that  $Q$  is nilpotent (which means there is some integer  $n > 0$  such that  $Q^n \equiv 0$ ), in particular  $Q^2 \equiv 0$ . Let  $\mathcal{H}$  be the full set of states of the theory, including ghosts and longitudinal photons and the like. Let  $\mathcal{H}_3$  be the kernel of  $Q$ , that is, all states  $|\psi_3\rangle$  in  $\mathcal{H}$  for which  $Q|\psi_3\rangle = 0$ . Let  $\mathcal{H}_2 = Q\mathcal{H}$ , that is, all states which are  $Q$  of something. As  $Q^2 = 0$ , all states in  $\mathcal{H}_2$  are killed by  $Q$  and thus in  $\mathcal{H}_3$ , so  $\mathcal{H}_2 \subset \mathcal{H}_3$ . Let  $\mathcal{H}_0 = \mathcal{H}_3/\mathcal{H}_2$  the coset space. Thus a state in  $\mathcal{H}_0$  is a state annihilated by  $Q$  modulo states which are  $Q$  of something. This is the cohomology of  $Q$ . It is also the set of physical states.

My objection to what P&S say is that their  $\mathcal{H}_1$  is not really a vector space (it doesn't contain 0, for instance).