Lecture 28 Dec. 10, 2007

GSW Electroweak Theory

Last time we considered a gauged symmetry group G and a scalar field which transforms by

$$\phi_i \to \left(e^{i\alpha^b\tau^b}\right)_{ij}\phi_j,$$

and assumed the symmetry is spontaneously broken so that $\langle\phi\rangle = \phi_0$. If the subgroup $K \subset G$ leaves ϕ_0 invariant, then in terms of the Lie algebras $\mathcal{K} \subset \mathcal{G}$ of K and G respectively, the coset space $\mathcal{G}/\mathcal{K}$ is the space that would have generated Goldstone bosons had the theory not been gauged, but instead this is the space of vector particles which develop masses, leaving massless only the gauge bosons belonging to $\mathcal{K}$. We considered three examples, two with $G = \mathrm{SU}(2)$ with either a complex doublet scalar or a real triplet scalar under the field. We found in the first case all the gauge particles became massive vector bosons, eating up all the Goldstone bosons, leaving only one massive real scalar, while in the second case only two of the gauge fields ate Goldstone bosons and became massive, leaving one massless gauge vector field and one massive real scalar.

We also introduced the full symmetry of a complex doublet scalar, $G = \mathrm{U}(2) = \mathrm{SU}(2) \times \mathrm{U}(1)$. Today we are going to discuss the application of this idea to explain the weak interactions. Historically the weak interactions were first seen in nuclear beta decay, which could be understood in terms of nucleon constituent decays, including the observed decay of a free neutron,

$$n \to p + e^- + \bar\nu_e.$$

Later when pions were discovered, their decay into muons and the subsequent decay of muons are also examples,

$$\begin{aligned}\pi^- &\to \mu^- + \bar\nu_\mu\\ \mu^- &\to e^- + \bar\nu_e + \nu_\mu\end{aligned}$$

Both the neutron and muon decays seem to be the result of a four-fermi interaction, but as we have seen, such an interaction term is highly non-renormalizable, as the coupling constant would need to have dimensions of

$[\mathrm{mass}]^{-2}$. Nonetheless this reaction was explored by considering the most general term,

$$\sum_i \bar\psi_1\Gamma_i\psi_2\bar\psi_3\Gamma_i\psi_4,$$

known as the Universal Fermi Interaction. It was observed that of the possible Γ's, that is, of the scalar, pseudoscalar, vector, pseudovector, and tensor possibilities, only the vector and pseudovector interactions were needed to fit the data. A big surprize, parity violation, meant there could be mixed Vector-Pseudovector terms, and in fact it was gradually realized that this was best described by a massive "Intermediate Vector Boson" which, in fact, was not a pure vector but "$V - A$", that is, coupling with $\gamma^\mu(1-\gamma_5)$.

Introducing this new vector particle improved the renormalization problem, as a $\bar\psi\gamma^\mu(1-\gamma_5)\psi A_\mu$ term in the lagrangian comes with a dimensionless coupling constant, but it was still unclear how to have a renormalizable theory of a massive vector particle. But with the Higgs mechanism we now see how to do that, because from the point of view of renormalizability, the symmetry breaking is irrelevant (it "goes away" at high energy energy, after all).

Note that in our representation, with

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0\end{pmatrix} \text{ and } \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1\end{pmatrix}, \quad \gamma^0\gamma^\mu(1-\gamma_5) = 2\begin{pmatrix}\bar\sigma^\mu & 0 \\ 0 & 0\end{pmatrix},$$

so only the upper, left handed, components of ψ and of $\psi^\dagger$ are coupled to the weak interactions. But of course both left and right handed electrons are negatively charged and couple to the photon (electromagnetism).

So today we consider a theory with gauge group $\mathrm{U}(2) = \mathrm{SU}(2) \times \mathrm{U}(1)$, where the component groups are called weak isospin and weak hypercharge. The gauge fields are $A^a_\mu, a = 1...3$ for the SU(2) part and B_μ for the U(1) part. Other particles will need to transform under a representation of SU(2), which we will always take to be either a doublet or singlet for weak isospin. For abelian groups, irreducible representations are always singlets, and each our fields will transform by multiplication by $e^{i\theta Y}$, where Y is a real number called the weak hypercharge of that multiplet. For fermions, the assignments of weak isospin and weak hypercharge must be done separately for the left-handed and right-handed components, ψ_L and ψ_R. In the kinetic term

$$\bar\psi i\partial\!\!\!/\psi = \bar\psi_L i\partial\!\!\!/\psi_L + \bar\psi_R i\partial\!\!\!/\psi_R,$$

the two derivatives will be covariantized using different representations of the $SU(2) \times U(1)$ group.

We will include a complex scalar doublet which transforms so that

$$D_\mu \phi = \left(\partial_\mu - igA^a_\mu \tau^a - \frac{i}{2} g' B_\mu \right) \phi,$$

which has two independent coupling constants because the group transformations cannot set the relative scales of the generators for A^a_μ relative to B_μ. In the language of representations, this means the scalar field ϕ is a doublet under $SU(2)$ and has weak hypercharge $Y = \frac{1}{2}$.

In addition to these fields, we need a whole bunch of fermions, for we are proposing the "standard model" which includes everything except gravity[1].

Read Peskin and Schroeder, pp 701-704

So we need to include three generations of leptons and three generations of doublets of quarks. Early on we made Dirac fields even though they were not irreducible representations of the proper isochronous Poincaré group, because we wanted symmetry under parity, but we now know that the weak interactions are not parity invariant, so we will consider the two pieces of a Dirac field, the left and right handed pieces, separately. The first generation spin 1/2 fields are

ψ	Name	Y	T_3	Q
$\nu_{e\,L}$	neutrino	$-1/2$	$1/2$	0
e^-_L	left handed electron	$-1/2$	$-1/2$	-1
u_L	left handed up quark	$1/6$	$1/2$	$2/3$
d_L	left handed down quark	$1/6$	$-1/2$	$-1/3$
e^-_R	right handed electron	-1	0	-1
u_R	right handed up quark	$2/3$	0	$2/3$
d_R	right handed down quark	$-1/3$	0	$-1/3$

and their antiparticles. There are also the second and third generations

$$\begin{pmatrix} \nu_{\mu\,L} \\ \mu^-_L \end{pmatrix}, \quad \mu^-_R, \quad \begin{pmatrix} \nu_{\tau\,L} \\ \tau^-_L \end{pmatrix}, \quad \tau^-_R, \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \quad \begin{pmatrix} c_R \\ s_R \end{pmatrix}, \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \quad \begin{pmatrix} t_R \\ b_R \end{pmatrix}.$$

[1] Well, the standard model also includes strong interactions mediated by the gluons, which are represented by another set of gauge fields without spontaneous symmetry breaking. There is no direct coupling between the gluons and the gauge fields A^a_μ and B_μ of the electroweak interaction. We will not discuss this color (QCD) interaction.

We have not included right handed neutrinos, because for the moment we are assuming neutrinos are massless and don't have right handed pieces. The Y, T_3 and Q quantum numbers of the second and third generation particles are the same as their first generation cousins, though their masses are much heavier (except, perhaps, for the neutrinos).

Read Peskin and Schroeder, pp. 704-705, 713-716.

Note for Eq. 20.101: If ψ transforms as a $T = \frac{1}{2}$ doublet, so $\psi \to e^{i\vec{\alpha}\cdot\vec{\sigma}/2}\psi$, the hermetian conjugate transforms by $\psi^\dagger \to \psi^\dagger e^{-i\vec{\alpha}\cdot\vec{\sigma}/2}$, as $\vec{\alpha}$ is real and the σ's are hermitian. Thus $\psi^\dagger\psi$ is a scalar. Both ψ and $\psi^\dagger$ transform under the spinor ($T = \frac{1}{2}$) representation of the isospin group SU(2), but under different presentations. If we want to make a scalar of two things which both transform like $\psi^\dagger$, we can define $\chi_a = \epsilon_{ab}\psi^\dagger_b$, where $\epsilon_{ab} = (i\sigma_2)_{ab}$ is the two-dimensional antisymmetric Levi-Civita tensor $\epsilon_{12} = 1 = -\epsilon_{21}$, $\epsilon_{11} = \epsilon_{22} = 0$ in isospin space. Then

$$\begin{aligned} \chi \quad \to \quad & i\,(\sigma_2)_{ab}\,\psi^\dagger_c \left(e^{-i\vec{\alpha}\cdot\vec{\sigma}/2}\right)_{cb} \\ & = i\,(\sigma_2)\, e^{-i\vec{\alpha}\cdot\vec{\sigma}^T/2}\psi^\dagger \\ & = e^{i\vec{\alpha}\cdot\vec{\sigma}/2} i\sigma_2\psi^\dagger = e^{i\vec{\alpha}\cdot\vec{\sigma}/2}\chi, \end{aligned}$$

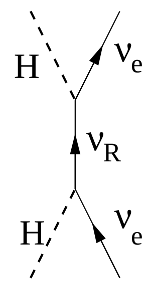
where in the last line we used $\sigma_2\vec{\sigma}^T\sigma_2^{-1} = -\vec{\sigma}$. So we see that χ transforms like ψ. In 20.101, $\chi = \epsilon^{ab}\bar{Q}_{La}$. [Note: we are not distinguishing between upper and lower isospin indices on the ϵ.]

Things get more complicated when we allow for three generations of leptons and quarks. As Rabi complained, "Who ordered that?" when the muon was identified as a heavy electron. With three generations of quarks, the left handed multiplet $Q_{L\,aj}$ develops a generation index j as well as an isospin index a, so the left-handed electron field is $Q_{L\,21}$. Now the most general quark-scalar coupling becomes

$$-\bar{Q}_{L\,ai}\phi_a M_{ij} D_{R\,j} - \epsilon^{ab}\bar{Q}_{L\,bi}\phi^\dagger_a N_{ij} U_{R\,j} + \text{h.c.}$$

with several 3×3 matrices of coupling constants. The off-diagonal elements mean that the one-particle eigenstates of energy are not the individual components of $Q_{L\,bi}$, but rather a mixture, as given in 20.105. If we reexpress our fields in terms of mass eigenstates, then the coupling to the weak vector bosons involve generation mixing, which permits transitions between different generations, such as the decay of strange particles, with the strange quark emitting a W^- and becoming a up quark, and the W^- then decaying into an down anti-up, or π^- meson, for example in $\Lambda \to \pi^- p$ decay.

On P. 715, things have changed. There is now very good evidence that there are neutrino oscillations and therefore that the observed neutrinos do have masses. Where could such a mass arise? In homework #4 (Peskin and Schroeder 3-4) we considered a mass term even for a Weyl neutrino fieldχ by adding a $\chi^T \sigma^2 \chi$ term to the Lagrangian. But our neutrinos are part of an isodoublet and have hypercharge, so invariance under G would require such a term to be coupled to something else, like the Higgs. In fact, $\chi = \epsilon_{ab} E_L^a \phi^b$ is a scalar under G and is $\sim \frac{v}{\sqrt{2}} \nu_e$ when ϕ takes the vacuum value, so coupling this to itself as a majorana mass term would give it mass. But this term has dimension $(\frac{3}{2}+1) \times 2 = 5$, so is nonrenormalizable and would have the enter the lagrangian with a coefficient $\frac{1}{M}$, for some mass M, which could be very large to explain why the observed neutrino masses are so small.



Where could such a term come from? We might imagine that at some very high mass scale there is some new physics that includes a heavy right handed Weyl netrino ν_R, and the term in the lagrangian might be $\bar{\nu}_R \chi$, with the above χ. This gives a $\bar{\nu}_R H \nu_e$ renormalizable coupling which at low energies would give an effective majorana coupling from χ^2, including a suppression factor from the mass in the ν_R propagator.

On page 716, experimental lower limits have gone up, claiming the Higgs mass must be greater than 114 GeV by direct search. But the best fits to various data which virtual Higgs can effect give a best fit for masses about 100 GeV. So we may be right on the edge of finding the Higgs.