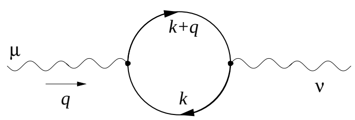
Lecture 23 Nov. 20, 2007

Renormalization of e, finite effects of $\Pi(q^2)$ Spontaneous Symmetry Breaking

We have been considering

$$i\Pi_2^{\mu\nu}(q) =$$



$$= \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\Pi(q^2),$$

and we have seen that the renormalized charge is related to the bare charge e_0 by

$$\alpha = \frac{e^2}{4\pi} = Z_3 \frac{e_0^2}{4\pi}, \quad \text{where } Z_3 = \frac{1}{\Pi(0)} \approx 1 - \frac{2\alpha}{3\pi\epsilon},$$

where $\epsilon = 4 - d$ will need to go to zero eventually. We also saw that we can define an effective coupling

$$\alpha_{\text{eff}}(q^2) = \frac{\alpha}{1 - \hat\Pi(q^2)},$$

where

$$\hat\Pi_2(q^2) := \Pi_2(q^2) - \Pi_2(0) = -\frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2}{m^2 - x(1-x)q^2}\right).$$

Today we will consider

- the behavior of $\text{Im}\,\hat\Pi(q^2)$ for timelike q^2.
- the behavior near $q^2 = 0$ in the nonrelativistic "Coulomb" potential, including a piece of the Lamb shift.
- the effect of the cut on $V(r)$ from near the threshold.
- the effective coupling constant $\alpha_{\text{eff}}(q^2)$ as a first example of a running coupling constant (renormalization group).

Read pp. 252–256

After having discussed these calculations of the first order loop effects, we should summarize what we have learned about renormalization and mention how this can be more satisfactorily understood.

Starting with the lagrange density

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar\psi\left(\partial\!\!\!/ - m_0\right)\psi - e_0\bar\psi\gamma^\mu\psi A_\mu,$$

we saw that we could calculate tree diagrams, the lowest order in perturbation theory, and these gave good results for physical processes. We could also calculate to higher order in perturbation theory, getting diagrams with loops, and these diagrams affect the measurements we make of the mass and charge of the electron, so that the real (measured) values are m and e, with

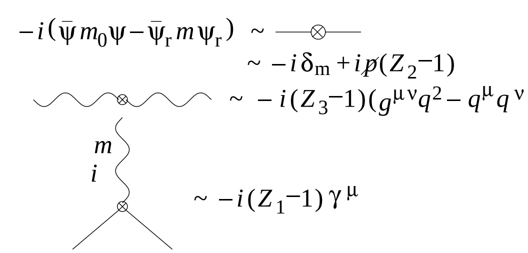
$$e = e_0 Z_3^{1/2}, \quad m = Z_2 m_0 - \delta m,$$

not the values e_0 and m_0 in the lagrangian, as we naïvely thought at the beginning. We also saw that $\bar\psi$ does not create a single electron with the expected amplitude but rather with $\sqrt{Z_2}$ of that strength. If we express physical processes in terms of the renormalized e and m, we get finite answers (except for experimentally unmeasurable infrared effects), but it is disturbing that we are perturbing about a Lagrangian infinitely far from what gives a good approximation.

To fix this, we should instead take as our unperturbed Lagrangian

$$\mathcal{L}_0 = -\frac{1}{4}F_r^{\mu\nu}F_{r\,\mu\nu} + i\bar\psi_r\left(\partial\!\!\!/ - m\right)\psi_r,$$

where we allow for the fields to change strength, $\psi = Z_2^{1/2}\psi_r$, $A^\mu = Z_3^{1/2}A_r^\mu$. Then our perturbation includes not only $-e\bar\psi_r\gamma_\mu\psi_r A_r^\mu$, but also "counterterms" such as $-\delta_m\bar\psi_r\psi_r$ to allow for not having included all of m_0 in the "unperturbed" lagrangian.



These counterterms all vanish to $0^{\rm th}$ order, but have $\mathcal{O}(e^2)$ terms. Dividing the $m_0\bar\psi\psi$ term into $m\bar\psi_r\psi_r$ and $\delta_m\bar\psi_r\psi_r$ is arbitrary, except that we impose a specific condition, that δ_m is chosen in each order of perturbation theory so that the pole comes out at $p^2 = m^2$, and the other divisions also satisfy conditions, $\Gamma^\mu(q^2{=}0) = \gamma^\mu$, $\Pi(q^2{=}0) = 1$, $\Sigma(\not p{=}m) = 0$ and $\left.\frac{d}{d\not p}\Sigma(\not p)\right|_{\not p=m} = 0$, where now the calculation of $\Pi(q^2)$ and $\Sigma(\not p)$ include not only the loop graphs but also the counterterms.

Seeing this worked out in detail, and showing that all calculations grouped so that the counterterms cancel the loop contributions at the specified values (renormalization points), and prevent all calculations from developing infinities at each order in perturbation theory, is a complex exposition and the focus of chapter 10. I will leave that for future studies by those of you who wish to continue as high energy theorists.

Phase Shift in the Course

Up to this point we have been pursuing an understanding of Quantum Field Theory, and though we have skipped some derivations and some of the sophisticated treatments, we have laid out the methods of perturbative QFT, so you can apply them to any theory relevant to your work. But we have not addressed any application more recent than the discovery of the structure of DNA ('53). There has been a lot of progress in high energy theory since then, but we will only have time to just touch on these developments in the next five lectures. These topics are

- spontaneous symmetry breaking (SSB)
- Yang-Mills (non-abelian gauge) field theories, and QCD
- the Higgs mechanism
- Electro-Weak theory

These are all essential ingredients of the Standard Model.

We will go on to these sexier issues. The first is spontaneous symmetry breaking.

Read pp. 347–352

We have only shown Goldstone's theorem in classical considerations, and you might wonder whether quantum corrections could give the Goldstone bosons masses, and also how we can even discuss ideas like being in the minimum of the potential in the full quantum theory. Section 11.2 discusses in detail the renormalization, in particular how maintaining the symmetry of the lagrangian when adding counterterms is possible and insures that the Goldstone bosons remain massless. Then in sections 3–5 the idea of an effective action in the presence of an external source is introduced. This is something like imposing an external magnetic field H on a system of spins, and finding the propagation of spins and thus the magnetization. The ground state in the presence of such a field (or source) is shifted, and the field (or spin) will in general develop a vacuum expectation value known as the "classical field", analogous to the magnetization. The effective action is a functional of the classical field, analogous to the Gibbs free energy.

Again this is very pretty but we don't have time to cover it. I leave it up to your future courses. Here condensed matter theorists may be especially interested.

Next time — Yang-Mills Non-Abelian Gauge Theory.