Lecture 18: November 4, 2013

Radiative Corrections; Classical Bremsstrahlung

When we discussed the general perturbation expansion we found that the Feynman diagrams in momentum space had 4-momentum conservation at each vertex, which put constraints on the momenta of internal lines (virtual particles). In some cases this conservation and the known momenta of all external lines is enough to determine the momenta of all internal lines as well. Indeed, this has been true for all the invariant amplitudes we have calculated so far. But when the momentum of a line is not so constrained, the momentum must be integrated. Such momenta flow through lines of the diagram in a loop, and these are called loop diagrams, as opposed to the ones we have seen so far, which are tree diagrams.

We will see that such diagrams often bring up new difficulties in interpretation of our Feynman rules, which will force us into a much more sophisticated exploration of the meaning of the fields and states and the parameters of the Lagrangian than we have done so far.

The problems we wish to address all come up when we look at the scattering of an electron by a charged source. That source could be another charged particle with its own complex interactions, but to simplify matters we will consider only a single interaction with this source, but three interactions of

\[ e \int d^3x \bar{\psi}(\vec{x})\gamma^\mu \psi(\vec{x}) A_\mu(\vec{x}) \text{ elsewhere.} \]

The scattering of an electron off the source will then include the diagrams of 6.1, in addition to the exchange of a single photon between the electron and the source. But it turns out we will not be able to make sense of the results unless we simultaneously consider another process, the emission of a single photon while the electron scatters, known as bremsstrahlung.

Those of you who took E&M II (504) with me studied Bremsstrahlung classically, from Jackson Chapter 15, and this classical understanding is essential to understand the quantum results. The reference to Jackson is page 709 in the 3rd edition, but the Peskin and Schroeder derivation is more intuitive. We will find that the probability of producing very low energy photons blows up, and we will need to address what this can mean, and how this complicates perturbation theory. We will see that this is also connected to the divergence of the loop momentum integral in the first diagram of 6.1.

Read pp 175-182

At the bottom of page 181, the lower limit chosen is unconvincing but gives a nice answer. We are trying to integrate over two spherical disks, one centered around \( \hat{v} \) and the other about \( \hat{v}' \). But things are done more properly in §6.4, so we can leave it till then, but if you are impatient, see “On Evaluating \( I(\vec{v}, \vec{v}') \)” in the supplementary notes.

You might also want to look at “Energy in Bremsstrahlung, Note on p. 179” in the supplementary notes.