Lecture 16 Oct. 29, 2007

$e^+ e^- \to \mu^+ \mu^-$; R; $e\,\mu$ Scattering

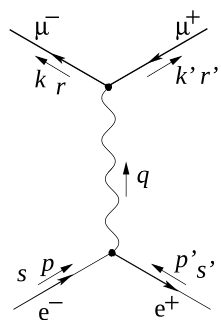
We have seen that the electron is described by a Dirac field ψ and the Lagrangian density for photons and electrons is

$$\mathcal{L} = \bar\psi\,(i\not\partial - m_e)\,\psi - e\bar\psi\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

The μ lepton behaves just like the electron, except for having a higher mass m_μ, so if we wish to describe μ's as well as electrons, we need to add a new field, which I will call Ψ, and add $\bar\Psi\,(i\not\partial - m_\mu)\,\Psi - e\bar\Psi\gamma^\nu\Psi A_\nu$ to $\mathcal{L}$. The propagator for the muon is the same as for the electron except for the value of the mass, and the charge and the interaction terms with photons the same.

Now let us consider $e^+ e^- \to \mu + \mu^-$:

The amplitude is given by



$$\begin{aligned}
&{}_0\langle k,r;k'r'|\,(-ie)\int d^4x\bar\Psi(x)\gamma^\rho\Psi(x)A_\rho(x) \\
&\qquad\times(-ie)\int d^4y\bar\psi(y)\gamma^\sigma\psi(x)A_\sigma(x)\,|ps;p's'\rangle_0 \\
&= \times 4\sqrt{E_pE_{p'}E_kE_{k'}} \\
&\qquad\times\langle 0|\,b^{r'}_{k'}a^r_k(-ie)\textstyle\int d^4x\bar\Psi(x)\gamma^\rho\Psi(x)A_\rho(x) \\
&\qquad\times(-ie)\displaystyle\int d^4y\bar\psi(y)\gamma^\sigma\psi(x)A_\sigma(x)a^{\dagger\,s}_p b^{\dagger\,s'}_{p'}\,|0\rangle \\
&= \bar v^{s'}_e(p')\,(-ie\gamma^\rho)\,u^s_e(p)\left(\frac{-ig_{\rho\sigma}}{q^2}\right)\bar u^r_\mu(k)\,(-ie\gamma^\sigma)\,v^{r'}_\mu(k').
\end{aligned}$$

Read Peskin and Schroeder pp. 131–141, 153–158

Note: On page 140, the book misdefines R, which should be the *ratio* of the cross section for $e^+e^- \to$ hadrons to the value we just calculated for $e^+e^- \to \mu^+\mu^-$, rather than the value of the cross section for $e^+e^- \to \mu^+\mu^-$. Note it is defined correctly in Figure 5.3.