

Lecture 12

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Wick's Theorem and Feynman Diagrams

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Last time we found that we could express any vacuum expectation value of the Time-ordered product of the full, interacting fields as

$$\langle \Omega | T \left(\prod_j \phi(x_j) \right) | \Omega \rangle = \lim_{T \rightarrow \infty(1-i\epsilon)} \frac{\langle 0 | T \left(\prod_j \phi_I(x_j) \right) e^{-i \int_{-T}^T dt H_I(t)} | 0 \rangle}{\langle 0 | T e^{-i \int_{-T}^T dt H_I(t)} | 0 \rangle}$$

Read Peskin and Schroeder, pages 88-99.

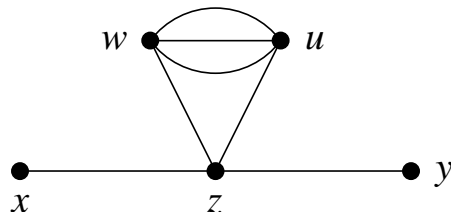
On page 92:

In order to do something different, let's consider a different piece in the contraction of

$$\langle 0 | \phi(x)\phi(y) \frac{1}{3!} (-i\mathcal{H}_I(z))(-i\mathcal{H}_I(w))(-i\mathcal{H}_I(u)) | 0 \rangle$$

given by

$$\frac{1}{3!} \left(\frac{-i\lambda}{4!} \right)^3 \langle 0 | \phi(x)\phi(y) \int dz \phi \phi \phi \phi \int dw \phi \phi \phi \phi \int du \phi \phi \phi \phi | 0 \rangle$$



For the contraction with $\phi(x)$, we get the same diagram if we contract with any of the four ϕ 's in any of the three internal vertices¹, for a factor of 12. Call that vertex the z vertex. Then $\phi(y)$ can be contracted with any of the three remaining ϕ 's on the z vertex² So we multiply by 3.

¹The other possibility, contracting with $\phi(y)$, would give a disconnected diagram (see p. 96) consisting of a noninteracting propagator times a product of disconnected pieces.

²Of course it could also be contracted with a different vertex, but that would not give rise to the same Feynman diagram. There are 8 equivalent ϕ 's for that possibility. Then there are several distinct ways the next ϕ on the z vertex can be contracted, several of which lead to the diagram explored in the book on p. 92.

Then the first of the remaining two ϕ 's from the z vertex can be contracted with any of the four ϕ 's in either of the remaining internal vertices³, so this gives us a factor of 8, and we choose to call the attached vertex w . The last ϕ on z must then be contracted with the remaining unattached vertex⁴, u to any of the 4 ϕ 's. There remains the three ϕ 's on the w vertex which can be attached to the three remaining ϕ 's on the u vertex in $3!$ ways⁵. So the symmetry factor for this diagram is

$$S = \frac{12 \times 3 \times 8 \times 4 \times 3!}{3! \times (4!)^3} = \frac{1}{12}.$$

³Or it could be contracted with the other z ϕ , which would give the first graph on page 93 multiplied by vacuum bubbles.

⁴Or to another of the w ϕ 's, in which case we could get other graphs not shown in the book.

⁵Or perhaps there is only one w - u connection, in which case the diagram looks like a mouse.