Faddeev-Popov ghosts

In evaluating the functional integral over gauge fields, we found that we could add a gauge-fixing term to the Lagrangian which would make the integrals better defined and evaluable in perturbation theory, but that in order to do so, a new factor appeared in the integrand. This new factor, the Faddeev-Popov determinant, is

$$\Delta^{-1}[A] = \det \frac{\delta \partial^\mu (A^\lambda)^{(b)}}{\delta \lambda^{(a)}}.$$  

We found that for Abelian gauge theory this factor is independent of all the fields and is therefore an ignorable constant. But this is not the case in non-Abelian theory. We now turn to evaluating this factor.

Recall that the Faddeev-Popov determinant is gauge invariant, so we can evaluate it for a field satisfying the Lorentz condition, $\partial^\mu A_\mu = 0$, and therefore the gauge transformations considered will be infinitesimal, and $(A^\lambda)^{(b)} = A^{(b)} + g^{-1}(D_\mu \lambda^{(b)})$. The Faddeev-Popov determinant is therefore the determinant of $g^{-1}$ times the operator $\partial^\mu D_\mu$, where the covariant derivative is to use the adjoint representation suitable for $\lambda$.