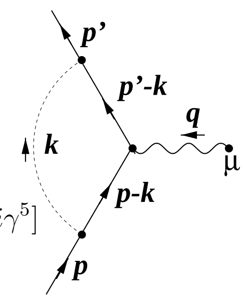
Physics 615 Due Nov. 15, 2007
Homework #10

1 [10pts] Do problem 6.3 from Peskin and Schroeder. This problem explores whether the measured values of $g-2$ places limits on the existence of possible postulated but undiscovered particles.

Solution 1 (a/c)

The diagram which describes the lowest order contribution of a scalar [or pseudoscalar] particle of mass M_h to the electron vertex is very similar to the photon correction. Its contribution is



$$\delta\Gamma^\mu(p',p) = -\frac{\lambda^2}{2}\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-M_h^2+i\epsilon} \times [i\gamma^5]\frac{i(\not p'-\not k+m)}{(p'-k)^2-m^2+i\epsilon}\gamma^\mu\frac{i(\not p-\not k+m)}{(p-k)^2-m^2+i\epsilon}[i\gamma^5]$$

where the $[i\gamma^5]$'s are to be included only for pseudoscalars.

The numerator is $i\lambda^2/2$ times

$$(\not p'-\not k+[-1]m)\gamma^\mu(\not p-\not k+[-1]m) \underset{\text{onshell}}{\longrightarrow} \begin{cases}(2m-\not k)\gamma^\mu(2m-\not k) & \text{for scalars}\\ \not k\gamma^\mu\not k & \text{for pseudoscalars}\end{cases}$$

Using $\not k\gamma^\mu\not k = 2k^\mu\not k - k^2\gamma^\mu$ in both cases, and $-2m\{\not k,\gamma^\mu\} = -4mk^\mu$ for scalars, we have

$$\begin{array}{ll} 4m^2\gamma^\mu - 4mk^\mu + 2k^\mu\not k - k^2\gamma^\mu & \text{for scalars}\\ 2k^\mu\not k - k^2\gamma^\mu & \text{for pseudoscalars.}\end{array}$$

The denominator is again written with Feynman parameters as

$$\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\frac{2}{D^3}$$

with

$$\begin{aligned} D &= x((p-k)^2-m^2)+y((p'-k)^2-m^2)+z(k^2-M_h^2)+(x+y+z)i\epsilon\\ &= k^2-2k\cdot(xp+yp')-zM_h^2+i\epsilon.\end{aligned}$$

With $\ell = k-xp-yp'$, and noting that $(p'-p)^2 = q^2 = 2(m^2-p'\cdot p)$ and thus $p'\cdot p = m^2-\frac{1}{2}q^2$, we have

$$\begin{aligned} D &= \ell^2-2xyp'\cdot p-(x^2+y^2)m^2-zM_h^2+i\epsilon\\ &= \ell^2+xyq^2-(x+y)^2m^2-zM_h^2+i\epsilon = \ell^2-\Delta+i\epsilon\end{aligned}$$

where $\Delta = -xyq^2+(1-z)^2m^2+zM_h^2$. Note that in a scattering process $q^2<0$ so Δ is positive. The numerators (in d dimensions) are $i\lambda^2/2$ times

$$\frac{2-d}{d}\ell^2\gamma^\mu + 2m(x+y)(xp^\mu+yp'^\mu) - \{(x+y)^2m^2-xyq^2\}\gamma^\mu + (1+[-1])4m(m\gamma^\mu - xp^\mu - yp'^\mu),$$

where the last term is for scalars only. The terms involving $2(xp^\mu+yp'^\mu) = (x+y)(p^\mu+p'^\mu)-(x-y)(p'^\mu-p^\mu)$ have a piece proportional to q^μ which vanishes, as it must for a photon coupling to an on-shell current, because it is antisymmetric under $x\leftrightarrow y$. Dropping these, and using the Gordon identity $p^\mu+p'^\mu \to -i\sigma^{\mu\nu}q_\nu + 2m\gamma^\mu$, the numerator is $i\lambda^2/2$ times

$$\left\{\frac{2-d}{d}\ell^2 + m^2(x+y)^2 + xyq^2 + 4m^2(1+[-1]) - 4m^2(x+y)(1+[-1])\right\}\gamma^\mu - 2m^2\left\{(x+y)^2 - 2(x+y)(1+[-1])\right\}\frac{i\sigma^{\mu\nu}q_\nu}{2m}.$$

We only need the δF_2 part,

$$\begin{aligned}\delta F_2(q^2) &= -2im^2\lambda^2\int\frac{d^d\ell}{(2\pi)^d}\int dx\,dy\,dz\,\delta(1-x-y-z) \times\frac{(x+y)^2-2(x+y)(1+[-1])}{(\ell^2-\Delta+i\epsilon)^3}\\ &= -2m^2\lambda^2\frac{S_4}{2(2\pi)^4}\frac{\Gamma(2)\Gamma(1)}{\Gamma(3)}\int dx\,dy\,dz\,\delta(1-x-y-z)\frac{\{(1-z)^2-2(1-z)(1+[-1])\}}{-xyq^2+(1-z)^2m^2+zM_h^2}\end{aligned}$$

For $q^2 = 0$, the x, y dependence is lost, and

$$\int dx\, dy\, \delta(1 - x - y - z) = \int_0^{1-z} dx = (1 - z),$$

so

$$\delta F_2(0) = \lambda^2 \frac{1}{16\pi^2} \int_0^1 dz \times \begin{cases} -\frac{(1-z)^3}{(1-z)^2 + Az} & \text{for pseudoscalars} \\ \frac{(1-z)^2(1+z)}{(1-z)^2+Az} & \text{for scalars} \end{cases},$$

where $A = M_h^2/m^2$. For $A \gg 1$, the dominant contribution is from small z, and we can drop the z's which don't multiply A in the integral, which becomes

$$\pm \int_0^1 dz\, \frac{1}{1 + Az} = \pm \frac{\ln A}{A}.$$

b For a Higgs of mass 60 GeV, and $\lambda = 3 \times 10^{-6}$, we have $A = 1.4 \times 10^{10}$, $\delta F_2 = 9.7 \times 10^{-23}$, way to little to see. For the muon, $A = 3.2 \times 10^5$, $\lambda = 6 \times 10^{-4}$, $\delta F_2 = 9 \times 10^{-14}$. Even though the experimental value is now better than the book says ($a_\mu = 1.0011659202$), still way to little to see.

For the axion case, we have the experimental constraint

$$10^{-10} > \frac{\lambda^2}{16\pi^2} f\left(\frac{m_A^2}{m_e^2}\right),$$

where

$$f(A) = \int_0^1 dx \frac{(1-x)^3}{(1-x)^2 + xA}.$$

In this case we are not insured of any experimental knowledge which enables us to look only at an asymptotic form. While f can be integrated by partial fractions, the result is rather messy. Clearly $f < 1$, so certainly we cannot exclude any $\lambda < 4\pi \times 10^{-5} \approx 10^{-4}$, regardless of the axion mass. Note that even for zero mass, there is no infrared blowup for the pseudoscalar correction.

As far as I can tell, neither the Higgs contribution nor the axion contribution are currently of interest, but there definitely is active work both on measuring and on calculating the anomolous magnetic moment. For example, see `http://arXiv.org/abs/hep-ph/0108192`, "The Standard Model Prediction for Muon g-2" and `http://arXiv.org/abs/hep-ex/0111046`, "Recent Progress on the BNL Muon g-2 Experiment".