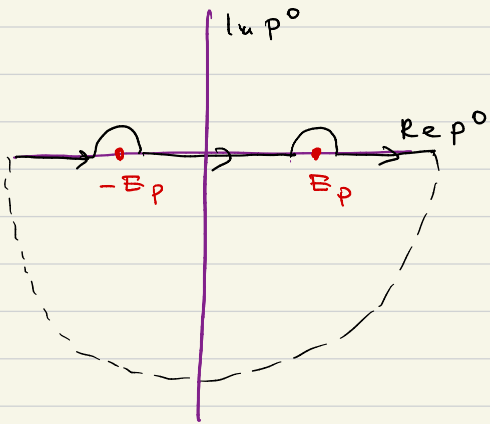
$$\langle[\phi(x),\phi(y)]\rangle = D(x-y) - D(y-x) = \int_{\vec{p}} \frac{e^{-i p\cdot(x-y)}}{2E_p}\Bigg|_{p^0=E_p} + \int_{\vec{p}} \frac{e^{-i p\cdot(x-y)}}{-2E_p}\Bigg|_{p^0=-E_p} =$$

$$\underset{x^0>y^0}{=} \int \frac{d^3p}{(2\pi)^3} \int \frac{dp^0}{2\pi i} \frac{-1}{p^2-m^2} e^{-i p\cdot(x-y)}$$

$$D(x-y) = \int_{\vec{p}} \frac{e^{-i p\cdot(x-y)}}{2E_p}$$

$$\shortparallel$$

$$\langle \phi(x)\phi(y)\rangle$$

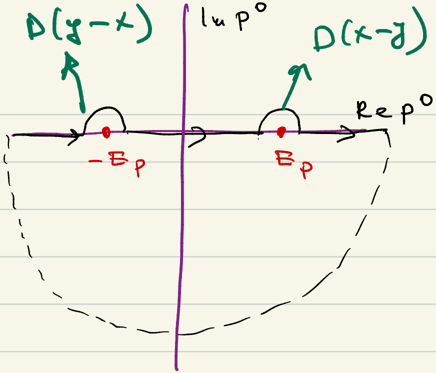


$$\oint \frac{dp^0}{2\pi i} \overset{f(p^0)}{\frac{-1}{p^2-m^2}} e^{-i p\cdot(x-y)} = 2\pi i \sum \text{res}$$

$$p^2 - m^2 = (p^0)^2 - |\vec{p}|^2 - m^2 = (p^0)^2 - E_p^2$$

$$f(p^0) = \frac{1}{(p^0-E_p)(p^0+E_p)}$$

$$f(z) \underset{z\to z_0}{\longrightarrow} \frac{\text{res}}{z-z_0}$$



$$I = \oint \frac{dp^0}{2\pi i} \overbrace{\frac{-1}{p^2 - m^2}}^{f(p^0)} e^{-ip\cdot(x-y)} \Rightarrow \sum \text{res}$$

$$p^2 - m^2 = (p^0)^2 - |\vec{p}|^2 - m^2 = (p^0)^2 - E_p^2$$

$$f(p^0) = \frac{1}{(p^0 - E_p)(p^0 + E_p)}$$

$$f(z) \xrightarrow[z \to z_0]{} \frac{\text{res}}{z - z_0}$$

$$I = \left.\frac{e^{-ip(x-y)}}{2E_p}\right|_{p^0 = E_p} + \left.\frac{e^{-ip(x-y)}}{-2E_p}\right|_{p^0 = -E_p}$$

$$D_R(x-y) = \Theta(x^0 - y^0) \langle [\varphi(x), \varphi(y)] \rangle =$$

$$= \Theta(x^0 - y^0) \langle \phi(x) \phi(y) \rangle + \Theta(x^0 - y^0) \langle \phi(y) \phi(x) \rangle$$

$$D_R(x-y) = \Theta(x^0-y^0) \langle [\varphi(x), \varphi(y)] \rangle$$

$$(\partial^2+m^2) D_R(x-y) = \partial^2 \Theta(x^0-y^0) \langle \cdots \rangle +$$

$$+ 2\, \partial_\mu \Theta(x^0-y^0) \langle [\partial^\mu \varphi(x), \varphi(y)] \rangle =$$

$$\int \delta'(x) f(x) = -\int \delta(x) f'(x) + \delta(x) f(x) \Big|_{-\infty}^{+\infty}$$

$$= - \delta(x^0-y^0) \langle [\frac{\partial \varphi(x)}{\partial x^0}, \varphi(y)] \rangle +$$

$$+ 2\, \delta(x^0-y^0) \langle \overset{\shortparallel}{\cdots} \rangle =$$

$$= -i\, \delta(x^0-y^0)\, \delta^{(3)}(\vec{x}-\vec{y}) = -i\, \delta^{(4)}(x-y)$$

$$\left\langle \left[\frac{\partial \varphi(x)}{\partial x^0}, \varphi(y) \right] \right\rangle = \left\langle \left[\pi(x), \varphi(y) \right] \right\rangle = -i\,\delta^{(3)}(\vec{x}-\vec{y})$$

$$(\partial^2+m^2)\, D_R(x-y) = -i\,\delta^{(4)}(x-y)$$

$$\hat{L}(x)\, G(x-x') = \delta(x-x')$$

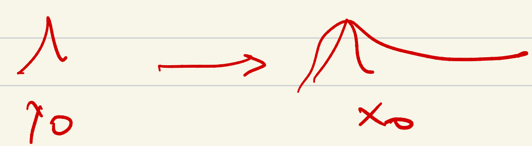
$D_R(x-y)$ - retarded Green's fn. of KG operator

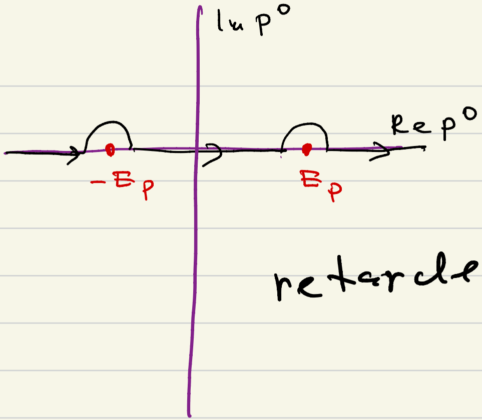
$$D_G(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}\, \tilde{D}_G(p)$$

$$(-p^2+m^2)\, \tilde{D}_G(p) = -i$$

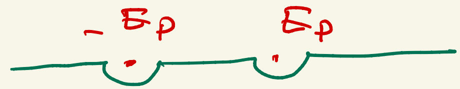
$$\tilde{D}_G(p) = \frac{i}{p^2-m^2}$$

$$D(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} e^{-ip(x-y)}$$

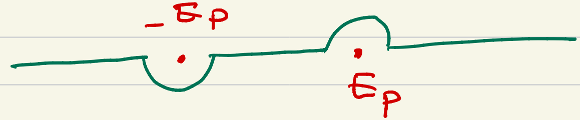




retarded



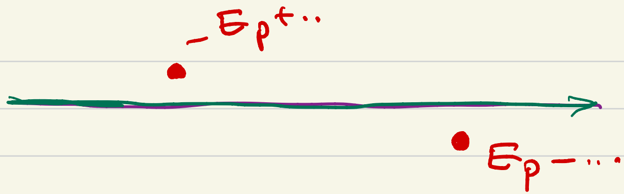
advanced

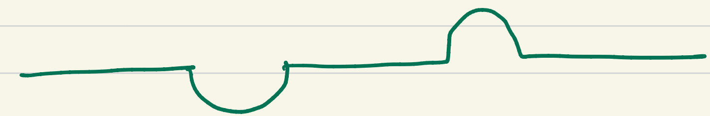


Feyman prescription

$$D_F(x-y) = \int_p \frac{i}{p^2-m^2+i\varepsilon} e^{-i p\cdot(x-y)}$$

$$(p^0)^2 - E_p^2 + i\varepsilon \approx (p^0)^2 - \left(E_p - \frac{i\varepsilon}{2E_p}\right)^2$$

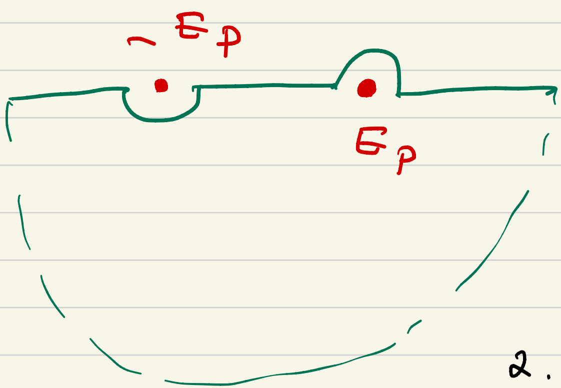




1. $x^0 > y^0$ $\quad e^{-ip\cdot(x-y)} \quad -ip^0(x^0-y^0)$

$$p^0 = u + iv$$

$$v(x^0-y^0)$$



$$D_F(x-y) = D(x-y) = \langle \phi(x)\phi(y)\rangle$$

2. $x^0 < y^0$ $\quad D_F(x-y) = D(y-x)$

$$= \langle \phi(y)\phi(x)\rangle$$

$$D_F(x-y) = \begin{cases} D(x-y), & x^0 > y^0 \\ D(y-x), & x^0 < y^0 \end{cases} =$$

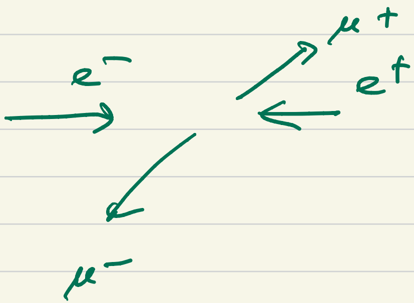
$$= \theta(x^0 - y^0)\langle \phi(x)\phi(y)\rangle + \theta(y^0 - x^0)\langle \phi(y)\phi(x)\rangle =$$

$$= \langle 0 | T\, \phi(x)\phi(y) | 0 \rangle$$

$$T\, \phi(x)\phi(y) = \begin{cases} \phi(x)\phi(y) & \text{if } x^0 > y^0 \\ \phi(y)\phi(x) & \text{if } y^0 > x^0 \end{cases}$$

$D_F(x-y)$ Feynman propagator for KG particle

$e^+e^- \to \mu^-\mu^+$

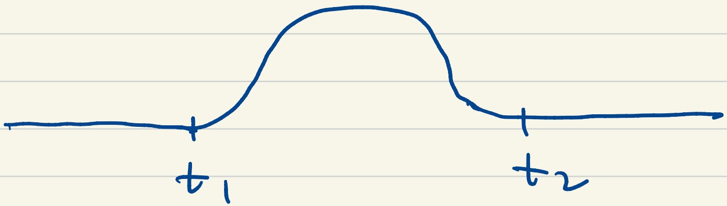


$$U = T\,e^{i\int H\,dt}$$

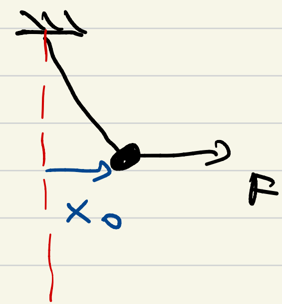
$$\psi(t_>) = U\;U\;U\;U\;\psi(t_<)$$

Particle creation by a classical source

$$(\partial^2 + m^2)\,\phi(x) = J(x)$$ ← source term



$$\ddot{\phi} + \omega^2 \phi = F$$



$$x \to x + x_0$$

$$\frac{kx^2}{2} + Fx$$

$$x \propto a + a^\dagger$$

$$a \to a + \alpha \qquad a^\dagger \to a^\dagger + \alpha^*$$

$$\langle (a^\dagger + \alpha^*)(a + \alpha) \rangle = |\alpha|^2 \qquad \Delta E = \omega |\alpha|^2$$

$$\mathcal{L} = \frac{(\partial_\mu \phi)^2}{2} - \frac{m^2 \phi^2}{2} + J(x)\,\phi(x)$$

$$a_p \to a_p + \dots \qquad \phi(x)$$