1 Physics 613: Problem Set 6 (due Friday April 22)

1.1 Attractive and repulsive QCD forces

Recall from class that if we scatter quark of color $i$ against anti-quark of color $k$, then the $t$-channel amplitude with color $j$ and $\ell$ in the final state is proportional to the color factor $T^a_{ji}T^a_{k\ell}$. Let’s re-examine this more properly, by considering the representations of $SU(3)_{\text{color}}$.

1. As we showed in HW5, the quark antiquark pair can either be in a singlet or an octet representation ($\bar{3} \otimes 3 = 8 \oplus 1$). Suppose the initial quark-antiquark state is in the singlet representation. Verify explicitly (you can use a computer, e.g. Mathematica or python) that the color factor is $T^a_{ji}T^a_{i\ell} = \frac{4}{3}\delta_{j\ell}$ (summation is implied on $a$ and $i$ here). So color is conserved (singlet to singlet) and the color force is attractive in the color singlet state.

2. Calculate the color factor for the initial state in the octet representation. (Think carefully about how to project onto the octet representation!!) Show that color is again conserved (octet to octet) and show that the color force is repulsive in the color octet state. [Hint: You will need the following identity $T^aT^bT^a = -\frac{1}{2N_c}T^b$.]

1.2 QCD theta term

Prove that the QCD theta term $\epsilon^{\mu\alpha\beta\gamma}\text{Tr}(F_{\mu\nu}F_{\alpha\beta})$ is gauge invariant and a total derivative. [Hint: you will need the following identity for the structure constants $f^{abc}f^{cde} = \frac{2}{N_c}(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) + d_{ace}d_{bde} - d_{bce}d_{ade}$ where $d$ is a totally symmetric tensor (known as the anomaly tensor).]

1.3 Running of $\alpha_s$ and the QCD scale

In class we discussed the one-loop running of the QCD fine structure constant:

$$\alpha_s(Q) = \frac{\alpha_s(0)}{1 + \frac{\alpha_s(0)}{4\pi} \beta \log \frac{Q^2}{m_0^2}}$$  \hspace{1cm} (1)

where

$$\beta = 11 - \frac{2}{3}N_f$$  \hspace{1cm} (2)

and $m_0$ is a UV reference scale where $\alpha_s = \alpha_s(0)$. Starting from $\alpha_s = 0.1$ at $m_0 = 1$ TeV, with $N_f = 6$, run the QCD coupling $\alpha_s$ down through each quark threshold to determine
\Lambda_{QCD} (the scale where \(\alpha_s\) appears to blow up at one-loop). You can assume at each quark mass threshold, all that changes is that \(\beta\) takes one smaller value of \(N_f\), and that \(\alpha_s\) is continuous through each threshold. You may also take \(m_t = 175\) GeV, \(m_b = 4\) GeV and \(m_c = 1.2\) GeV.